

### **Section 3**

## **Assessing the CAHSEE Mathematics Standards**

The Mathematics part of the California High School Exit Examination (CAHSEE) assesses designated California content standards from grades 6 and 7 and Algebra 1. A multiple-choice format is used to assess six strands: Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; Measurement and Geometry; Algebra 1; and Mathematical Reasoning. Each of these strands is described in detail in the following section of the Teacher Guide. For reporting purposes, the Statistics, Data Analysis, and Probability strands for grades 6 and 7 are combined. The Mathematical Reasoning test questions, which are always based on concepts in the other five strands, are reported under those strands. Thus there is no reporting category specifically for Mathematical Reasoning.

The CAHSEE is focused on mathematics constructs that are taught and assessed throughout elementary, middle, and high school. Three underlying constructs have been identified for the mathematics part of the CAHSEE: computational and procedural skills, conceptual understanding, and problem solving. Test questions on the exam cover one or more of these constructs, and CAHSEE test question writers and reviewers verify that each question measures the appropriate construct as well as the identified content standard. The constructs for each standard are given in the following pages.

Although test questions for the mathematics part of the CAHSEE do not specifically test students on mathematics vocabulary, they may require students to understand mathematical terms. It is especially important that students know the terms that appear in the language of the content standards associated with a question.

The following pages of the Teacher Guide discuss the mathematics strands and content standards included in the CAHSEE. The mathematics strands are:

- Number Sense
- Statistics, Data Analysis, and Probability
- Algebra and Functions
- Measurement and Geometry
- Algebra 1
- Mathematical Reasoning

After each strand is described, each standard in that strand is discussed in detail, and a sample released test question is provided to illustrate each standard. Also included are possible reasons that students might select the distractors, as well as an explanation of the correct answer. Teachers will find this section of the guide useful in understanding how the California content standards are assessed on the CAHSEE. A thorough understanding of the standards and the test questions associated with them will help teachers focus their instruction on the content standards and better prepare students for the exam.

## **The Number Sense Strand**

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Students' understanding of fractions, decimals, percents, and integers—and their relationship to each other and to the other disciplines of mathematics—is an essential component of their mathematics learning. CAHSEE test questions in the Number Sense strand require students to demonstrate a foundational understanding of numbers and ways they are represented.

Students will be asked to:

- solve problems with fractions, decimals, and percents
- compare and order numbers
- demonstrate an understanding of percents, including those less than 1 and greater than 100
- use ratios and proportions
- understand and meaningfully interpret large and small numbers in scientific notation
- use specific characteristics of numbers, such as multiples, factors, and primes
- use and represent integers as the basis for the comparison of quantities

Essential to success in this CAHSEE strand is the student's understanding of the mathematical operations and the ways they are related to each other. This understanding includes:

- the meaning of arithmetic operations with fractions, decimals, and integers
- the associative and commutative properties of addition and multiplication

- the distributive property of multiplication over addition
- the understanding and use of inverse relationships of addition and subtraction, multiplication, and division
- finding square roots, squaring numbers, and using the inverse relationship between them

Students also should possess computational fluency. They should be able to select appropriate methods and tools for computing with fractions and decimals; perform mental arithmetic; use algorithms for computing with fractions, decimals, and integers; use strategies for estimation and for judging the reasonableness of results; and be able to analyze and explain methods for solving problems with proportions.

The 10 California content standards covered by the CAHSEE Number Sense strand are discussed in the following pages.

Strand            **Number Sense (NS)**

 Standard        **NS1.1**

**Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.**

 Construct        **Conceptual Understanding**

**The radius of the earth's orbit is 150,000,000,000 meters. What is this number in scientific notation?**

**A**     $1.5 \times 10^{-11}$

**B**     $1.5 \times 10^{11}$

**C**     $15 \times 10^{10}$

**D**     $150 \times 10^9$

Scientific notation is required knowledge in science and engineering because many numbers are either so large or so close to zero that there is no other convenient way to write them. CAHSEE test questions in this standard require students to demonstrate understanding of the basic concepts of scientific notation using approximations of very large and very small numbers. Test questions may also involve the translation of approximate numbers into scientific notation, the comparison of numbers in scientific notation with either positive or negative exponents, and the understanding of the relative size of two numbers in scientific notation.

### *Sample Test Question*

The sample question gives the radius of the earth's orbit as 150,000,000,000 meters and asks students to translate that number into scientific notation. The correct answer is choice B. Students should recognize that the place-value distance from the 1 (highest place value, 100 billion) to the decimal is 11 digits and that this value provides an appropriate representation of the equivalence as  $10^{11}$ , also equivalent to  $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ . Students also should know that expressions in scientific notation may include a multiplier, between 1 and 10, along with an exponential value of 10. While exact powers of 10 are expressed without a multiplier (e.g.,  $10^{11}$ ), numbers such as 150,000,000,000 require a multiplier along with the equivalent power of 10. A typical method of finding the multiplier and the exponent is to count the number of decimal places the decimal must move to create a number between 1 and 10. In the example, the decimal point is moved 11 places to the left to get 1.5 for the multiplier and + 11 for the exponent.

### *Analysis of Distractors*

Two of the distractors in the sample test question, C and D, represent common errors students may make in finding equivalence between standard and scientific notation: Both distractors are equivalent in number to 150,000,000,000 (and  $1.5 \times 10^{11}$ ), but neither has a multiplier within the appropriate range (from 1 to 10). Distractor A expresses the power of 10 as  $\frac{1}{10^{11}}$  (until students have a clear understanding of the magnitude of very large

and very small numbers, they are often unsure which direction the decimal point should move.)

Strand	Number Sense (NS)	<b>Which of the following numerical expressions results in a negative number?</b>  <b>A</b> $(-7) + (-3)$ <b>B</b> $(-3) + (7)$ <b>C</b> $(3) + (7)$ <b>D</b> $(3) + (-7) + (11)$
Standard	NS1.2	
<b>Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.</b>		
Constructs	<b>Procedural Skills,</b> <b>Conceptual</b> <b>Understanding</b>	

All students should understand the basic arithmetic functions involving rational numbers in all forms and be comfortable in performing calculations with positive and negative numbers. CAHSEE test questions in this standard require students to demonstrate computational fluency with rational numbers and an understanding of the relationships between these types of numbers. These skills are fundamental to achievement of the California content standards in mathematics.

### *Sample Test Question*

The sample question asks students to compute four combinations of values with different signs and then identify which of the sums is negative. The correct answer is choice A. Students should understand that adding rational numbers with unlike signs requires finding the difference of their absolute values and then choosing the sign of the addend with the greater absolute value. Students should also recognize that adding rational numbers with like signs requires adding the absolute values and keeping the sign, as in choice A  $(-7) + (-3) = -10$ .

### *Analysis of Distractors*

The distractors present students with errors in computing with integers. Two of the distractors, B and C, have the same absolute values as the addends in the correct answer, A, with sums of 4 and 10 respectively, but they are both positive integers. Distractor D introduces a third value and has a sum of 7, but it too is a positive integer.

Strand	Number Sense (NS)	<b>Some students attend school for 180 of the 365 days in a year. About what percentage of the year do they attend school?</b>  <b>A    18%</b> <b>B    50%</b> <b>C    75%</b> <b>D    180%</b>
Standard	NS1.3 Convert fractions to decimals and percents and use these representations in estimations, computations and applications.	
Constructs	Procedural Skills, Conceptual Understanding	

Students using mathematics in their daily lives will need to know how to convert decimals to fractions to percents with ease. The recognition of equivalent forms is essential for student fluency with numbers. CAHSEE test questions in this standard require students to demonstrate facility in finding equivalent values and representations for numbers as well as computing values using fractions, decimals, and percents. Students must also be able to use these various representations for estimating and performing computations in mathematical applications.

### *Sample Test Question*

The sample question asks students to determine what percent 180 is of 365. The correct answer is choice B. Students should recognize that the correct value is found by dividing the part (180) by the whole (365), giving  $180 \div 365 \approx 0.493$ . The question also requires conversion of 0.493 to a percent by multiplying by 100 ( $0.493 = 49.3\%$ ). Finally, students should recognize that the phrase “about what percentage” in the stem calls for a rounded percent rather than an exact value, and that  $49.3\% \approx 50\%$ . It is to be expected that many students will use mental estimation skills, rather than computation, to determine that 180 is about half of 365.

### *Analysis of Distractors*

Two of the distractors, A and D, represent a misunderstanding of 180 as 18% and 180%.

Distractor C indicates incorrect computation of the decimal equivalent of  $\frac{180}{365}$  or a possible guess with apparent plausibility.

Strand	Number Sense (NS)	<p>The cost of an afternoon movie ticket last year was \$4.00. This year an afternoon movie ticket costs \$5.00. What is the percent increase of the ticket from last year to this year?</p> <p>A 10%</p> <p>B 20%</p> <p>C 25%</p> <p>D 40%</p>
Standard	NS1.6	
Constructs	Procedural Skills, Conceptual Understanding	
	Calculate the percentage of increases and decreases of a quantity.	

CAHSEE test questions in this standard require students to demonstrate understanding of percent increase and decrease, which is a fundamental tool in analyzing numerical information. For example, a price change of one dollar can be very meaningful in terms of buying a loaf of bread and inconsequential in terms of buying a car. Students should understand that percent change clarifies the impact of this kind of change, and they should be able to calculate the change with facility. Standard NS1.6 will also be assessed with test questions that require students to find the percent decrease, or what quantity would result from a given percentage of increase of an original quantity.

### Sample Test Question

The sample question requires students to calculate the percent of increase for a movie ticket whose price increased from \$4.00 to \$5.00. The correct answer is choice C. Students should understand that finding the percent increase or decrease of a quantity requires first finding the difference between the initial value and the final value. In the sample question, the difference is \$1.00. Then to find the percent increase, students must know to compare the difference to the initial cost by using division:  $\frac{\$1.00}{\$4.00}$ . Finally, the resulting decimal must be converted to its equivalent percent:  $1 \div 4 = 0.25 = 25\%$ .

### Analysis of Distractors

The distractors represent errors resulting from a misunderstanding of the concept being tested. Distractor A results from taking the difference between the two ticket prices and converting it to a percent. Distractor B results from multiplication of the two values in the problem, \$4.00 and \$5.00. Distractor D results from converting the original ticket price to a percent.



Strand	<b>Number Sense (NS)</b>	<b>Sally puts \$200.00 in a bank account. Each year the account earns 8% simple interest. How much interest will be earned in three years?</b>  <b>A    \$16.00</b> <b>B    \$24.00</b> <b>C    \$48.00</b> <b>D    \$160.00</b>
Standard	<b>NS1.7</b>	
<b>Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.</b>		
Constructs	<b>Procedural Skills, Conceptual Understanding, Problem Solving</b>	

CAHSEE test questions in this standard require students to solve a variety of problems involving percents. Both consumers and people working in business need to understand the mathematical meaning of common business terms such as commission and profit interest computations. Solving problems of these types is one of the most important skills students need as they become adults. Understanding these concepts and their applications can mean the difference between students managing their money and other resources well, or not at all. This standard will be also be assessed with test questions that require students to find simple and compound interest, as well as discounts, markups, and commissions. A maximum of three iterations is used for questions that involve calculating compound interest. The iterations include the initial multiplication of principal by interest rate.

### *Sample Test Question*

The sample question asks students to determine the amount of simple interest \$200 will earn in three years at the given rate. The correct answer is choice C. Students should recognize that simple interest is calculated by multiplying the principal by the annual rate and then multiplying by the time. In the sample question, the principal is \$200, the rate is 8%, and the time is 3 years. To calculate correctly, students are also required to convert 8% to its decimal equivalent ( $\$200 \times .08 \times 3 = \$48$ ).

### *Analysis of Distractors*

The distractors represent errors resulting from failure to perform one of the required steps and/or from a computation error. Distractor A represents one year's interest and results from the multiplication of the principal and the interest rate only. Distractor B represents multiplication of the rate times the number of years only. Distractor D may be attractive to students who converted the interest rate to a decimal incorrectly, multiplying \$200 by .8 and also failed to multiply by the number of years.

Strand            **Number Sense (NS)**

Standard        **NS2.1**

**Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.**

Constructs      **Procedural Skills,  
Conceptual  
Understanding**

$$\frac{10^{-2}}{10^{-4}} =$$

A     $10^{-6}$

B     $10^{-2}$

C     $10^2$

D     $10^8$

CAHSEE test questions in this standard require students to understand the concept of negative exponents. One of the most powerful concepts in mathematics is that compact means of notation can be extended to include new concepts. Negative exponents are an example of this kind of extension.

### *Sample Test Question*

The sample question presents a numerical calculation requiring students to demonstrate their understanding of the rule for dividing expressions involving exponents with a common base. The correct answer is choice C. Students should understand that  $10^{-2}$  is

equivalent to  $\frac{1}{10^2}$  and  $\frac{1}{10^{-4}}$  is equivalent to  $10^4$ . Thus it is possible to represent the

problem as  $10 \cdot 10 \cdot 10 \cdot 10 / 10 \cdot 10$ , making the underlying concept more apparent. Once the students master the concept behind negative exponents, they understand that when dividing exponential expressions with the same base, the exponents must be subtracted,

so that  $\frac{10^{-2}}{10^{-4}}$  is equivalent to  $10^{(-2)-(-4)}$ , which is equivalent to  $10^2$ .

### *Analysis of Distractors*

The distractors represent misunderstandings of the concepts involved in the calculation.

Distractor A represents the addition of the exponents or an error in the subtraction of  $(-4) - (-2)$ . Distractor B shows a failure to apply the negative signs correctly in the same subtraction. Distractor D represents a failure to understand the fundamental concept, as it results from multiplying  $-2$  by  $-4$ .

Strand	<b>Number Sense (NS)</b>	<p>Which of the following is the prime factored form of the lowest common denominator of <math>\frac{7}{10} + \frac{8}{15}</math>?</p> <p>A <math>5 \times 1</math></p> <p>B <math>2 \times 3 \times 5</math></p> <p>C <math>2 \times 5 \times 3 \times 5</math></p> <p>D <math>10 \times 15</math></p>
Standard	<b>NS2.2</b>	
	<b>Add and subtract fractions by using factoring to find common denominators.</b>	
Constructs	<b>Procedural Skills, Conceptual Understanding</b>	

The focus of this content standard is on the students' ability to add and subtract fractions with unlike denominators that share one or more factors. Students should be able to find the prime factorization of the lowest common denominator of two whole numbers and to factor as they attempt to find a common denominator. CAHSEE test questions in this standard require students to perform addition and subtraction arithmetic using equivalent fractions with common denominators. The algorithmic approach of this standard is associated with the requirement that common denominators be determined by factoring.

### *Sample Test Question*

The sample question requires students to find the needed common denominator for 10 and 15 using the prime factor technique for finding common denominators. The correct answer is choice B. Students should recognize that the prime factors for 10 are  $2 \times 5$  and that the prime factors for 15 are  $3 \times 5$ . Since the common prime factor is 5, including the additional factors of 2 and 3 gives  $2 \times 3 \times 5$  as the prime factors of the least common denominator.

### *Analysis of Distractors*

The distractors represent misunderstandings of the concept being tested. Distractor A shows 5 as a common multiple only, without the additional factors required. Distractor C incorrectly repeats 5 as a factor of the common denominator, indicating a failure to see that this value is common to both fractions. Distractor D shows the denominators multiplied by each other, as students will often use this method to find a common denominator.

Strand	<b>Number Sense (NS)</b>	$(3^8)^2 =$  <b>A</b> $3^4$ <b>B</b> $3^6$ <b>C</b> $3^{10}$ <b>D</b> $3^{16}$
Standard	<b>NS2.3</b>	
	<b>Multiply, divide, and simplify rational numbers by using exponent rules.</b>	
Constructs	<b>Procedural Skills, Conceptual Understanding</b>	

CAHSEE test questions in this standard require students to select the appropriate rules for operations with exponents with common bases and perform accurate computations in simplifying rational numbers. Students should understand the following rules:

- adding exponents when multiplying numbers with common bases
- subtracting exponents when dividing numbers with common bases
- multiplying exponents when raising a number to a particular power

Test questions may include those requiring multi-step operations, such as the simplification of numerators and denominators with common factors.

### *Sample Test Question*

The sample question requires students to expand  $(3^8)^2$  using the rule for multiplying exponents in parentheses  $[(a^b)^c = a^{bc}]$ . The correct answer is choice D. In this instance, students should simply use the rule to determine that  $(3^8)^2 = 3^{16}$ .

### *Analysis of Distractors*

The distractors present the other three operations that could be performed. Distractor A represents the inappropriate operation of division, rather than multiplication. Distractor B represents the inappropriate operation of subtraction. Distractor C represents the addition of the exponents, which students might choose if they confused this calculation with one requiring multiplication of exponential expressions with the same base.

<p>Strand            <b>Number Sense (NS)</b></p> <p>Standard        <b>NS2.4</b>  <b>Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.</b></p> <p>Constructs      <b>Procedural Skills, Conceptual Understanding, Problem Solving</b></p>	<p><b>The square root of 150 is between</b></p> <p><b>A</b> 10 and 11.  <b>B</b> 11 and 12.  <b>C</b> 12 and 13.  <b>D</b> 13 and 14.</p>
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CAHSEE test questions in this standard require students to demonstrate a conceptual understanding of powers and roots and their inverse relationship. The idea of mathematical inverse is a key precursor for algebraic reasoning, and students should understand that roots are the inverse of raising a number or expression to a power. For students should know that  $\sqrt{9^2} = 9$ . Students will not be required to calculate the example, square root for a number that is not a perfect square, but students should be able to approximate the value of the square root of an integer that is not a perfect square.

### *Sample Test Question*

The sample question asks students to find an approximation of the square root of 150, an integer that is not a perfect square. The correct answer is choice C. Students should recognize that they must first determine both the closest perfect square greater than the given integer and the closest perfect square less than the integer. This task may involve some trial and error multiplication along with the application of knowledge of squares and square roots. Since the number 150 is not a perfect square, students may recall or calculate that 144 is a perfect square and that 169 is the next perfect square. Since 144 is close to but less than 150 and 169 is close to but greater than 150, the square root of 150 must lie between those two perfect squares.

### *Analysis of Distractors*

The distractors misplace 150 between other numbers and may be selected by students who do not understand the concept of square root or who may incorrectly calculate the square of one of the numbers.

Strand	Number Sense (NS)	<b>If <math> x  = 3</math>, what is the value of <math>x</math>?</b>  <b>A</b> $-3$ or $0$ <b>B</b> $-3$ or $3$ <b>C</b> $0$ or $3$ <b>D</b> $-9$ or $9$
Standard	NS2.5	
<b>Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.</b>		
Constructs	<b>Procedural Skills, Conceptual Understanding, Problem Solving</b>	

CAHSEE test questions in this standard require students to demonstrate a conceptual understanding of absolute value and its meaning as represented on a number line. Relating absolute value to distance on the number line may help students understand the concept: Distance cannot be negative, but there will always be two numbers on the number line that are the same distance from zero. Questions may require students to find the absolute value after performing a basic computation.

### *Sample Test Question*

The test question asks students to determine the possible values for  $x$  in a simple absolute value equation. The correct answer is choice B. Students should recognize that since the absolute value of a number is the distance on a number line from that number to zero in either direction, all absolute values are positive numbers:  $|x| = x$  and  $|-x| = x$ . In the test question, if  $|x| = 3$ , then  $x = -3$  or  $x = 3$ , since  $|3| = 3$  and  $|-3| = 3$ .

### *Analysis of Distractors*

The distractors represent misunderstandings of the concept and notation for absolute value. Distractors A and C incorrectly equate the absolute value of 3 with 0 and offer either a negative or a positive value of 3. Distractor D correctly provides both a negative and a positive value but inappropriately associates the absolute value of 3 with the square of 3.

## **The Statistics, Data Analysis, and Probability Strand**

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To demonstrate knowledge and skills in the Statistics, Data Analysis, and Probability strand, students must understand the fundamental concepts involved in data collection, display, and analysis. Students will be asked to determine ways to collect, organize, and display relevant data to answer questions, formulate questions that can be addressed with data, select and use appropriate statistical methods to analyze data, and develop and evaluate inferences and predictions that are based on data. In addition, students are required to understand and apply the basic concepts of probability.

Specifically, the standards in the Statistics, Data Analysis and Probability strand include the following knowledge and skills:

- finding measures of central tendency to characterize data
- interpreting and evaluating conclusions based on data
- organizing and representing possible outcomes for events and expressing theoretical probabilities
- representing probabilities as ratios, proportions, and percents
- understanding the numerical continuum of probability between impossibility (0) and absolute certainty (1)
- recognizing the difference between independent and dependent events
- displaying data appropriately, including both one- and two-variable data sets
- understanding and computing quartiles

The eight California content standards covered by the CAHSEE Statistics, Data Analysis, and Probability strand are discussed in the following pages.

Strand	<b>Statistics, Data Analysis, and Probability (P)</b>	<b>Rico's first three test scores in biology were 65, 90, and 73. What was his mean score?</b>  <b>A</b> 65 <b>B</b> 73 <b>C</b> 76 <b>D</b> 90
Standard	<b>6P1.1</b>	
	<b>Compute the mean, median, and mode of data sets.</b>	
Constructs	<b>Procedural Skills, Conceptual Understanding</b>	

One of the major objectives of the Statistics, Data Analysis, and Probability strand is to give students tools to help them understand the uses and misuses of statistics. This CAHSEE content standard has three components: computation of the mean, computation of the median, and recognition of the mode of data sets. Statistical measures of central tendency represent important methods for summarizing and comparing single-variable data sets. Students should understand the significance of each as a measure of central tendency as well as the differences among these measures. While most students are familiar with calculating the mean (average) from dealing with grades, they should also be able to find the median of a data set and identify the mode. For this standard, students will not be asked to find the median of an even number of values.

Students should know that

- the median is the middle score of an ordered set of numbers, where half the scores are greater than the median and half are less
- the mode is the number which appears more frequently
- the mean is most affected by extreme values

### *Sample Test Question*

The sample question gives the data set (65, 90, 73) and asks students to compute the mean. The correct answer is choice C. Students should recognize that they should compute the mean by first finding the sum ( $65 + 90 + 73 = 228$ ) and then dividing by 3 ( $228 \div 3 = 76$ ).

### *Analysis of Distractors*

The distractors represent conceptual misunderstandings about measures of central tendency. Distractor A is the minimum of the data set; distractor B is the median of the data set; and distractor D is the maximum of the data set.



Strand                    **Statistics, Data  
Analysis, and  
Probability (P)**

Standard            **6P2.5**  
**Identify claims based on  
statistical data and, in simple  
cases, evaluate the validity of the  
claims.**

Constructs           **Conceptual  
Understanding,  
Problem Solving**

**Three-fourths of the 36 members of a club  
attended a meeting. Ten of those attending  
the meeting were female. Which one of the  
following questions can be answered with  
the information given?**

- A**    How many males are in the club?
- B**    How many females are in the club?
- C**    How many male members of the club  
attended the meeting?
- D**    How many female members of the club  
did not attend the meeting?

This content standard has two components: identifying claims made on the basis of statistical data and evaluating the validity of the claims based on statistical data. Because students should be able to understand statistical claims as well as they understand purely verbal arguments for or against a position, students should develop skills to evaluate the quality of data and conclusions based on data. CAHSEE test questions for this standard may ask students to identify a valid claim based on data or to recognize a question for which the data could be used to provide an answer.

### *Sample Test Question*

The sample test question presents data about members of a club: Three fourths of the members, or 27 of 36, attended a meeting, and 10 were female. The answer choices represent four possible questions that these data might be used to answer. The correct answer is choice C. Students should recognize that the information given is mostly about those attending the meeting, and little is known about the members who did not attend. Thus C is the correct response because the number of male members attending the meeting can be calculated by subtracting the number of females in attendance from the total ( $27 - 10 = 17$ ).

### *Analysis of Distractors*

The questions presented in distractors A and D cannot be answered, based on the data, because it is not known how many of the members who did not attend (one fourth of 36) are male or female. Although distractor B might look like an attractive choice because, like the stem of the question, it deals with females, it also cannot be answered with the given information: Although it is known how many females attended the meeting, there is no information about the number of females who were not at the meeting.

**Strand**                      **Statistics, Data Analysis, and Probability (P)**  
  
**Standard**                **6P3.1**  
**Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.**  
  
**Constructs**                **Conceptual Understanding, Problem Solving**

To get home from work, Curtis must get on one of the three highways that leave the city. He then has a choice of four different roads that lead to his house. In the diagram below, each letter represents a highway, and each number represents a road.

		Highway		
		A	B	C
Road	1	A 1	B 1	C 1
	2	A 2	B 2	C 2
	3	A 3	B 3	C 3
	4	A 4	B 4	C 4

If Curtis randomly chooses a route to travel home, what is the probability that he will travel Highway B and Road 4?

- A  $\frac{1}{16}$
- B  $\frac{1}{12}$
- C  $\frac{1}{4}$
- D  $\frac{1}{3}$

Organizing structures such as sample spaces, diagrams, and tables are useful for the representations of probabilities, and the ability to create a structured representation of a complex situation is an important reasoning tool. To demonstrate achievement on this standard, students must recognize appropriate and correct representations of events. From the correct representation, they must derive an understanding of the relationship between the frequency of the outcome and its numerical expression. They should be able to determine a theoretical probability of any particular outcome based on a correct representation.

*Sample Test Question*

The sample question includes a diagram that represents combinations of route choices (3 highways, 4 roads) and asks students to determine the probability of an individual randomly taking a given route. The correct answer is choice B. Students should recognize that if there are  $n$  possible outcomes for an independent event and  $r$  possible outcomes for another independent event, there are  $nr$  outcomes for the two events together. In other words, if there are  $n$  ways to do one thing and  $r$  ways to do another thing, there are  $nr$  ways to do the two things together. Thus three highways  $\times$  four roads  $= 3 \times 4 = 12$  routes, and the probability of selecting any individual route  $= \frac{1}{12}$ .

*Analysis of Distractors*

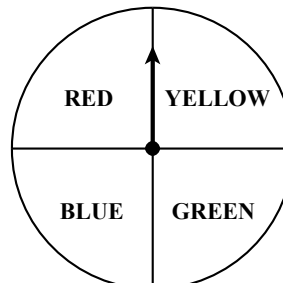
The distractors represent misunderstandings of the ways to determine probability and/or failure to complete the steps in the problem. Distractor A represents an inappropriate squaring of the probability of taking one road. Distractor C represents the probability of taking any one road, and distractor D represents the probability of taking any one highway.

Strand            **Statistics, Data  
Analysis, and  
Probability (P)**

Standard        **6P3.3**  
**Represent probabilities as ratios,  
proportions, decimals between 0  
and 1, and percentages between 0  
and 100, and verify that the  
probabilities computed are  
reasonable; know that if P is the  
probability of an event, 1-P is the  
probability of an event not  
occurring.**

Constructs       **Conceptual  
Understanding,  
Problem Solving**

**What is the probability that the spinner  
will NOT stop on red if it is spun one  
time?**



- A  $\frac{1}{4}$
- B  $\frac{1}{3}$
- C  $\frac{3}{4}$
- D  $\frac{4}{3}$

All students should understand that mathematical probability is used to predict what might happen in the future and that probabilities are ratios determined by considering the likely results or outcomes of events. CAHSEE test questions for this standard cover all of the components of the standard. Students are expected to:

- know that probabilities are ratios that can be expressed as fractions, decimals, or percentages
- compute the probability of a described event
- verify the reasonableness of a computed probability
- compute the probability that an event will not occur

*Sample Test Question*

The sample question asks students to determine the probability that an event will not occur—that a spinner will not land on one quadrant of a circle. The correct answer is choice C. Students should recognize that because the four regions covered by the spinner have equal areas, the probability of the spinner stopping on any one region is  $\frac{1}{4}$ .

Therefore the probability that it will **not** stop on a given region is  $1 - \frac{1}{4}$ , or  $\frac{3}{4}$ .

*Analysis of Distractors*

The distractors represent misconceptions about the probability of an event not occurring. Distractor A gives the probability that the event will occur; this value has not been subtracted from 1. Distractor B represents a misconception that only three quadrants should be used to calculate the probability and gives the probability of the spinner landing on one of the three. Distractor D represents either of two possibilities: using a fraction to represent the total number of quadrants divided by three quadrants (all but red) or obtaining the correct answer but then inverting it.

Strand            **Statistics, Data  
Analysis, and  
Probability (P)**

Standard        **6P3.5**  
**Understand the difference  
between independent and  
dependent events.**

Construct       **Conceptual  
Understanding**

A bag contained four green balls, three red balls, and two purple balls. Jason removed one purple ball from the bag and did not put the ball back in the bag. He then randomly removed another ball from the bag. What is the probability that the second ball Jason removed was purple?

A     $\frac{1}{36}$

B     $\frac{1}{9}$

C     $\frac{1}{8}$

D     $\frac{2}{9}$

Discerning the difference between dependent and independent events is important in evaluating probabilistic outcomes. CAHSEE test questions in this content standard require students to understand that events are independent of each other if the occurrence or non-occurrence of one event does not affect the probability of the occurrence or non-occurrence of another event. Similarly, students must recognize that events are dependent if the occurrence or non-occurrence of one event affects the probability of the occurrence or non-occurrence of another event. Computation may be required to determine the result of the independent or dependent events.

### *Sample Test Question*

The test question asks students to demonstrate understanding of the probability of the occurrence of a dependent event. The correct answer is choice C. Students should determine that initially the bag contains 4 green, 3 red, and 2 purple balls, for a total of 9 balls, and that when 1 purple ball is removed and not replaced, the bag contains a total of 8 balls. Since there are now 8 balls with only 1 being purple, the probability of randomly choosing the purple ball is  $\frac{1}{8}$ .

### *Analysis of Distractors*

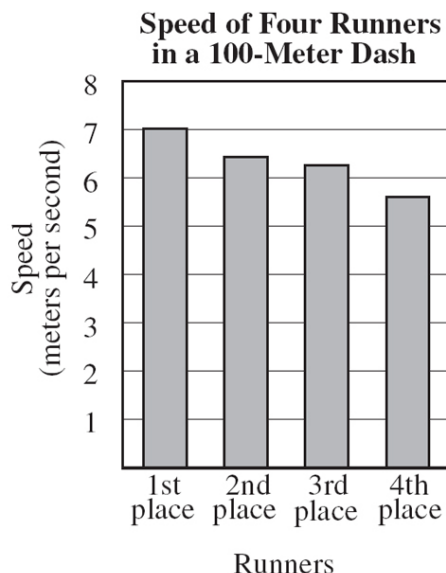
The distractors offer misunderstandings of the underlying concepts in the problem.

Distractor A results from the inappropriate multiplication of  $\frac{2}{9} \times \frac{1}{8}$ , which is the probability of randomly choosing both purple balls. Distractor B results from computing the probability of choosing 1 purple ball from 9 balls. Distractor D results from the addition of two probabilities for independent events:  $\frac{1}{9} + \frac{1}{9}$ .

Strand **Statistics, Data Analysis, and Probability (P)**

Standard **7P1.1**  
**Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.**

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



**Based on the bar graph shown above, which of the following conclusions is true?**

- A** Everyone ran faster than 6 meters per second.
- B** The best possible rate for the 100-meter dash is 5 meters per second.
- C** The first-place runner was four times as fast as the fourth-place runner.
- D** The second-place and third-place runners were closest in time to one another.

Large data sets are difficult to grasp mentally without an accessible visual representation. CAHSEE test questions in this standard require students to recognize and interpret various forms of display and to compare two sets of data displayed the same way. The forms of display for single-variable data sets assessed on the CAHSEE include bar graphs, line graphs, pictograms, stem and leaf plots, box and whisker plots, and circle graphs. Students should be able to determine subsets of the following: median and mode, minimum and maximum, upper and lower quartiles, and a comparison of percentages in a whole. CAHSEE test questions for this standard may also require students to select an appropriate type of data display.

### *Sample Test Question*

The sample question presents a bar graph that shows the average speed, in meters per second (m/s), of four runners over a 100-meter distance and asks students to identify an accurate conclusion supported by the data. The correct answer is choice D. Students should use the graph to determine the speed of each runner and then evaluate and

compare the four answer choices. From the graph, the first-place runner's speed was approximately 7.1 m/s; the second place runner's speed was approximately 6.5 m/s; the third place runner's speed was approximately 6.3 m/s; and the fourth place runner's speed was approximately 5.6 m/s. This analysis shows that the second place and third place runners' times were closest together.

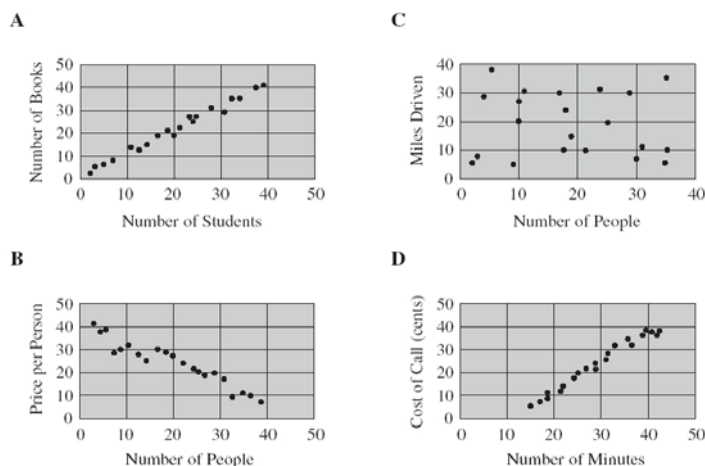
*Analysis of Distractors*

Students should recognize that distractor A is incorrect because the fourth place runner ran at approximately 5.6 m/s; distractor B is incorrect because all four runners ran faster than 5 m/s; and distractor C is incorrect because 7.1 m/s is not four times faster than 5.6 m/s.



Strand	<b>Statistics, Data Analysis, and Probability (P)</b>
Standard	<b>7P1.2</b>
	<b>Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).</b>
Constructs	<b>Conceptual Understanding Problem Solving</b>

Which scatter plot shows a negative correlation?



The identification of patterns and relationships, including clustering and trends, as well as the concept of correlation (positive, negative, or none) are significant aspects of using data. Students should understand correlation as a measure of the relationship between two variables, with negative correlation as the association of an increase in the value of one variable with a decrease in the corresponding value of the second variable. Students should also recognize that positive correlation is the association of an increase in the value of one variable with an increase in the corresponding value of the second variable.

CAHSEE test questions for this standard address the following components of the standard:

- representing two variables on a scatterplot
- determining the distribution of the variables
- recognizing the apparent relationships between the two variables represented

### *Sample Test Question*

The sample item requires students to identify the graph that illustrates a negative correlation. The correct answer is choice B: As the price per person increases, the number of people decreases, indicating a negative correlation.

### *Analysis of Distractors*

The distractors are plots that show either a positive correlation or no correlation. Distractor A represents a positive correlation: As the number of books increases, the number of students increases. Distractor C indicates no correlation: As miles driven increases, the number of people both increases and decreases over the range of miles driven. Distractor D indicates a positive correlation: As the cost of one call increases, the number of minutes increases (at or greater than values of 15).

Strand	<b>Statistics, Data Analysis, and Probability (P)</b>	<b>Joel's scores on eight English quizzes are 11, 12, 14, 15, 17, 18, 20, and 21. What is the upper quartile value of the scores?</b>
Standard	<b>7P1.3 Understand the meaning of, and be able to compute the minimum, the lower quartile [value], the median, the upper quartile [value], and the maximum of a data set.</b>	<b>A 18</b> <b>B 19</b> <b>C 20</b> <b>D 21</b>
Constructs	<b>Procedural Skills, Conceptual Understanding</b>	

Certain data sets require the interpretation of one value in relation to the entire data set. If the values of a data set are arranged from lowest to highest, the minimum, lower quartile, maximum and upper quartile divide the data into four groups that are approximately the same size. This provides useful and meaningful representation of data. CAHSEE test questions for this standard require students to compute and identify, for various lists of data, the following specific data characteristics:

- minimum
- lower quartile
- median
- upper quartile
- maximum

Students should understand that if the values are in random order, they must first be ordered before calculating or identifying a specific value. Students must also be able to interpret these values from graphical representations such as stem-and-leaf plots or bar graphs.

### *Sample Test Question*

The sample question provides a data set and asks students to determine the upper quartile value. The correct answer is choice B. Students should understand that the upper quartile is the median of the top half of the values in the data set. Since there are 8 values in all, there are 4 in the upper half: 17, 18, 20, and 21. Therefore the upper quartile is the mean of the 2 middle values (18 and 20).

### *Analysis of Distractors*

The distractors represent misunderstandings of the concept of quartiles. Distractors A and C may be attractive to students who assume that a median or quartile value must be present within the data set. Distractor D, the maximum, will be attractive to students who incorrectly associate the upper quartile with maximum value.

## The Algebra and Functions Strand

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The Algebra and Functions strand is most closely associated with the representation of quantitative relationships as a style of mathematical thinking for formalizing patterns, functions, and generalizations. As students increase their mathematical knowledge and skills, they work frequently with algebraic symbols, expressions with variables, and graphical representations. It is essential that students develop an understanding of several different meanings and uses of variables through multiple representations. Everyday experiences with linear functions should aid in the development of the concepts of proportionality and the ability to discriminate between linear and nonlinear functions. Students must also learn to recognize and generate equivalent expressions, solve linear equations, and effectively use formulas.

To demonstrate achievement in this strand, students will be asked to:

- work with patterns and relationships
- represent, analyze, and generalize a variety of patterns with table, graphs, and symbolic rules
- compare different forms of representations
- identify functions
- use algebraic expressions
- solve linear equations

The use of mathematical models to represent and understand quantitative relationships is developed by modeling and solving contextualized problems. The analysis of change in various contexts involves tools such as graphs to analyze the nature of changes in quantities in linear relationships.

The 10 California Content Standards covered by the CAHSEE grade 7 Algebra and Functions strand are discussed in the following pages.

Strand	<b>Algebra and Functions (AF)</b>	<b>Which of the following inequalities represents the statement, “A number, <math>x</math>, decreased by 13 is less than or equal to 39”?</b>  <b>A</b> $13 - x \geq 39$ <b>B</b> $13 - x \leq 39$ <b>C</b> $x - 13 \leq 39$ <b>D</b> $x - 13 < 39$
Standard	<b>AF1.1</b>	
	<b>Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).</b>	
Constructs	<b>Procedural Skills, Conceptual Understanding, Problem Solving</b>	

Translating verbal descriptions into mathematical expressions is essential in solving real-world problems. CAHSEE test questions in this standard require students to translate between verbal descriptions and mathematical equivalents. Students should be able to use variables and appropriate operations to write or identify an expression, an equation, a system of equations or an inequality to solve a problem. Test questions may also ask students to set up an appropriate equation.

### *Sample Test Question*

The sample question requires students to translate a verbal description of an inequality into a mathematical expression. The correct answer is choice C. Students should recognize that “a number,  $x$ , decreased by 13” is represented as  $x - 13$  and that “less than or equal to 39” is represented by  $\leq 39$ . Putting both parts of the statement together,  $x - 13 \leq 39$ .

### *Analysis of Distractors*

The distractors offer expressions that use the same values found in the problem but represent verbal expressions not given in the stem. Distractor A represents “13 decreased by a number,  $x$ ,” and also “greater than or equal to 39.” Distractor B uses the correct inequality notation but, like distractor A, represents “13 decreased by a number,  $x$ .” Distractor D presents the appropriate expression for “a number,  $x$ , decreased by 13” but represents “less than 39,” rather than “less than or equal to 39.”

Strand	<b>Algebra and Functions (AF)</b>	If $h = 3$ and $k = 4$ , then $\frac{hk + 4}{2} - 2 =$  <b>A</b> 6 <b>B</b> 7 <b>C</b> 8 <b>D</b> 10
Standard	<b>AF1.2</b>	
	<b>Use the correct order of operations to evaluate algebraic expressions such as <math>3(2x + 5)^2</math></b>	
Constructs	<b>Procedural Skills, Conceptual Understanding</b>	

CAHSEE test questions for this standard require students to select and use the correct order of arithmetic operations in evaluating expressions (parentheses, exponents, multiplication, division, addition, subtraction). Students may also be required to evaluate expressions that include the distributive property and other basic properties of real numbers.

### *Sample Test Question*

The sample question asks students to evaluate an expression using the correct order of operations. The correct answer is choice A. Students should first multiply  $h$  by  $k$ , add 4, divide by 2, and then subtract 2:  $\frac{(3)(4)+4}{2} - 2 = \frac{12+4}{2} - 2 = \frac{16}{2} - 2 = 8 - 2 = 6$ .

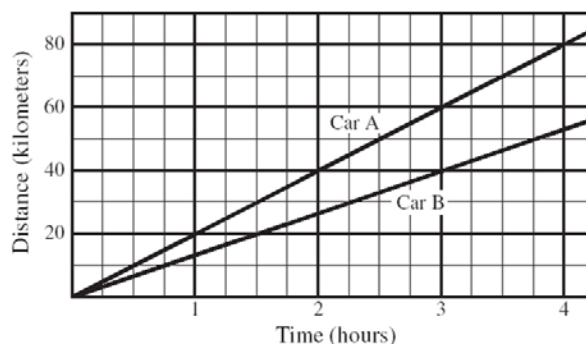
### *Analysis of Distractors*

The distractors offer solutions that result from performing operations in improper order. Distractor B is obtained by dividing by 2 before evaluating the numerator. Distractor C is obtained by dividing the product of 3 and 4 by 2 before evaluating the numerator. Distractor D is obtained by adding 4 before multiplying 3 by 4.

Strand **Algebra and Functions (AF)**

Standard **AF1.5**  
**Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.**

Constructs **Conceptual Understanding, Problem Solving**



After three hours of travel, Car A is about how many kilometers ahead of Car B?

- A 2
- B 10
- C 20
- D 25

CAHSEE test questions for this standard focus on either of its two main components. The first component requires the selection and execution of a graph that accurately and appropriately represents a quantitative relationship. The second component requires the interpretation and/or alternate representation of information presented in graphical form.

### *Sample Test Question*

The sample question presents a graph showing distance traveled over time for two cars, A and B, and asks for a specific interpretation of the information shown in the graph. The correct answer is choice C. Students should recognize that distance, in kilometers, is recorded on the y-axis, and time, in hours, is recorded on the x-axis. Three hours on the time scale corresponds to 60 kilometers for Car A, and three hours corresponds to 40 kilometers for Car B. The number of kilometers that Car A is ahead of Car B after 3 hours is represented by the difference between the distance traveled by Car A and the distance traveled by Car B in the same time (60 kilometers – 40 kilometers = 20 kilometers).

### *Analysis of Distractors*

The distractors represent misreadings of the graph. Distractor A gives the number of hours that Car A has traveled when it has gone 40 kilometers, the distance Car B traveled in 3 hours. Distractor B represents an error in reading the scale of the graph, assuming that the increments have a value of 10. Distractor D represents use of the approximate difference in distance at 4 hours, rather than 3.

Strand            **Algebra and  
Functions (AF)**

Standard        **AF2.1**  
**Interpret positive whole-number  
powers as repeated multiplication  
and negative whole-number  
powers as repeated division or  
multiplication by the  
multiplicative inverse. Simplify  
and evaluate expressions that  
include exponents.**

Constructs       **Procedural Skills,  
Conceptual  
Understanding**

$$x^3 y^3 =$$

- A  $9xy$
- B  $(xy)^6$
- C  $3xy$
- D  $xxxyyy$

This standard has four main components: the concept of positive whole-number powers as repeated multiplication, the concept of negative whole-number powers as repeated division, multiplication by the multiplicative inverse, and simplification and evaluation of expressions that include exponents. The first two components of this standard are assessed in the related Number Sense standards 2.1 and 2.3. CAHSEE test questions that assess student achievement in this standard may require students to evaluate monomial expressions. Other questions for this standard may require students to demonstrate an understanding of the multiplicative inverse.

### *Sample Test Question*

The sample question asks students to interpret as repeated multiplication the algebraic expression for  $x$  raised to the third power multiplied by  $y$  raised to the third power. The correct answer is choice D. Students should recognize that  $x^3 = x \cdot x \cdot x$  and that  $y^3 = y \cdot y \cdot y$ , so that  $x^3 y^3 = x \cdot x \cdot x \cdot y \cdot y \cdot y = xxxyyy$ .

### *Analysis of Distractors*

The distractors represent misunderstandings of the meaning of the exponents and/or the appropriate operation. Distractor A multiplies the exponents and uses the product as a coefficient. Distractor B adds the exponents, as would be appropriate for like, rather than unlike, variables. In distractor C, the value of the exponents has been moved to serve as a coefficient.

Strand	<b>Algebra and Functions (AF)</b>	<p><b>Simplify the expression shown below.</b></p> $(6a^4bc)(7ab^3c)$ <p> <b>A</b> <math>13a^4b^3c</math>  <b>B</b> <math>13a^5b^4c^2</math>  <b>C</b> <math>42a^4b^3c</math>  <b>D</b> <math>42a^5b^4c^2</math> </p>
Standard	<b>AF2.2</b>	
<b>Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.</b>		
Construct	<b>Conceptual Understanding</b>	

CAHSEE test questions in this standard require students to multiply and divide monomials of the form  $Ax^n$  and to expand powers and find roots to monomials of the same form whose roots have integer exponents. Items may also include combinations of multiplying and dividing monomials. Students should be comfortable with the rules for multiplying and dividing exponential expressions with the same base.

### *Sample Test Question*

The sample question requires students to demonstrate knowledge of the rules for multiplying monomials, multiplying the numeric values and adding the exponents. The correct answer is choice D, as  $6 \times 7 = 42$  and the product of  $a^4$  and  $a^1$  is  $a^5$ ; the product of  $b^1$  and  $b^3$  is  $b^4$ ; and the product of  $c^1$  and  $c^1$  is  $c^2$ .

### *Analysis of Distractors*

The distractors represent errors in multiplying the numeric values and/or the exponents. Distractor A represents the incorrect operation of adding, rather than multiplying, 6 and 7 and also the failure to use  $a$  as  $a^1$ ,  $b$  as  $b^1$ , and  $c$  as  $c^1$  when adding the values of the exponents. Distractor B represents the incorrect value obtained by adding 6 and 7 combined with the correct values for the exponents. Distractor C represents the correct multiplication of the numeric values but, like distractor A, has the incorrect multiplication of the exponents  $a$ ,  $b$ , and  $c$ .

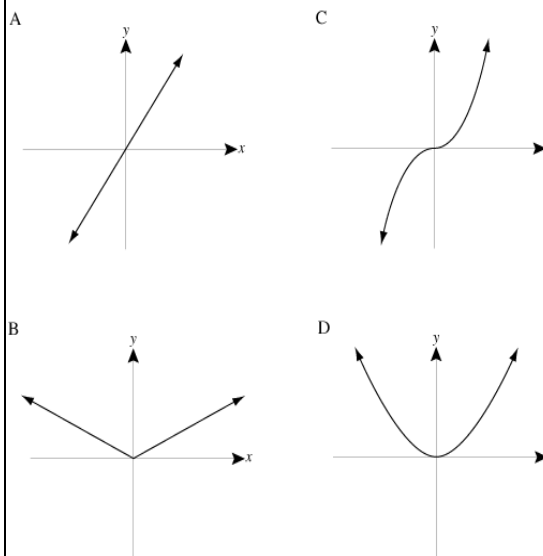


Strand **Algebra and Functions (AF)**

Standard **AF3.1**  
**Graph functions of the form  $y = nx^2$  and  $y = nx^3$  and use in solving problems.**

Construct **Conceptual Understanding**

**Which of the following could be the graph of  $y = x^3$ ?**



CAHSEE test questions in this standard require knowledge of graphing functions, as demonstrated by selecting the appropriate graph of a given function or by selecting the appropriate function for a given graph. Some questions may also require knowledge of function graphing to solve problems. As part of their foundational understanding of functions, students should be able to predict the shape of a graph based on the characteristics of the given function (e.g., linear, quadratic).

### *Sample Test Question*

The sample question asks students to identify which graph could represent the function  $y = x^3$ . The correct answer is choice C. Students should understand the basic concepts underlying the problem—that cubic functions are nonlinear and that negative values for  $x$  correspond to negative values for  $y$  and positive values for  $x$  correspond to positive values for  $y$ .

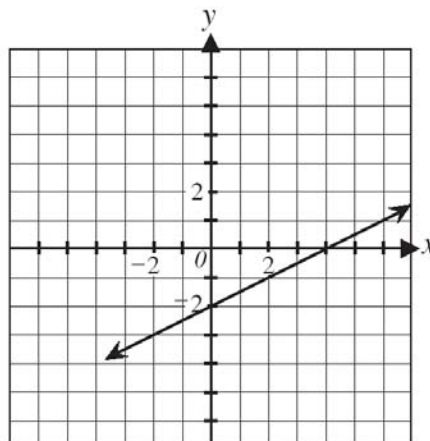
### *Analysis of Distractors*

Distractors A and B are graphs of linear, rather than nonlinear, functions, and distractor D represents a quadratic function in which negative values for  $x$  correspond to positive values for  $y$ .

Strand **Algebra and Functions (AF)**

Standard **AF3.3**  
**Graph linear functions, noting that the vertical change (change in  $y$ -value) per unit of horizontal change (change in  $x$ -value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.**

Constructs **Procedural Skills, Conceptual Understanding**



What is the slope of the line shown in the graph above?

- A -2
- B  $-\frac{1}{2}$
- C  $\frac{1}{2}$
- D 2

Students should understand that linear functions can model many real-world phenomena and that the rate of change in a function is shown by the slope of the graph of the function. A conceptual understanding of slope can be a key element in students' development of proportional reasoning skills. CAHSEE test questions for this standard may focus on either of its two main components. The first component is graphing linear functions on the  $xy$  coordinate system. The second is the identification of the slope in quantitative terms from a given linear function or the selection of a given slope from a numerical value, from a line shown on a graph, or from two pairs of coordinate points.

### *Sample Test Question*

The sample question shows a graph of a linear function and asks students to determine the slope represented by a line that crosses the  $y$ -axis at the point  $(0, -2)$  and the  $x$ -axis at the point  $(4, 0)$ . The correct answer is choice C. Students should understand slope as the change in  $y$  divided by the change in  $x$  and/or as the ratio "rise over run." In this problem, the change in the  $y$ -value is obtained by subtracting  $-2$  from  $0$ , and the change in the  $x$ -value is obtained by subtracting  $0$  from  $4$ , and thus

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{0 - (-2)}{4 - 0} = \frac{2}{4} = \frac{1}{2}.$$

*Analysis of Distractors*

Distractor A is simply the value of the  $y$ -intercept and represents a misunderstanding of the concept of slope. Distractor B represents an error in the subtraction of  $0 - (-2)$  or  $4 - 0$ . Distractor D represents the error of dividing the change in  $x$  by the change in  $y$ .

**Strand Algebra and Functions (AF)**

**Standard AF3.4**

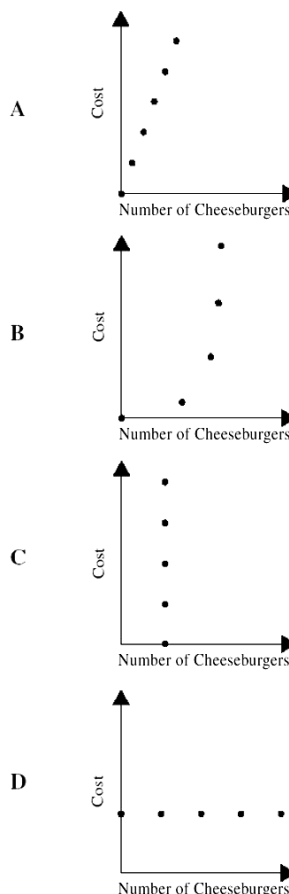
**Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of a line equals the [ratio of the] quantities.**

**Construct Conceptual Understanding**

10. Best Burger sells cheeseburgers for \$1.75 each. Part of a table representing the number of cheeseburgers purchased and their cost is shown below.

Number Purchased	Cost(\$)
0	0
1	1.75
2	3.50
3	5.25
4	7.00

Which of the following is a portion of the graph of the data in the table?



Graphing direct variation is a powerful way to comprehend and express proportional reasoning. CAHSEE test questions within this standard focus on either of its two main components, both of which require students to understand the relationship between the graphical presentation of data and the symbolic representation of data. The first component involves the identification of the correct graph. The second component involves the determination of the slope of a direct variation and the interpretation of the meaning of the slope as a constant ratio between the two quantities in the variation.

*Sample Test Question*

The sample question presents a table of values showing the number of cheeseburgers purchased and the corresponding cost for each additional cheeseburger. Students are asked to select the appropriate graph to represent the data. The correct answer is choice A. Students must understand the relationship between the number of cheeseburgers on the  $x$ -axis and the corresponding cost for each additional cheeseburger on the  $y$ -axis. The table shows that each additional cheeseburger purchased increases the total cost by \$1.75. In other words, students should recognize that the relationship is linear: As the value on the  $x$ -axis increases by 1, the value on the  $y$ -axis increases correspondingly by \$1.75.

*Analysis of Distractors*

The distractors are graphs that incorrectly represent the function in the table. Distractor B shows a nonlinear relationship between number and cost. Cost increases exponentially, so that following purchase of cheeseburger number three, it has risen higher than the \$7.00 indicated in the table. Distractor C represents a constantly increasing cost without the purchase of additional cheeseburgers after the first one. Distractor D represents the purchase of additional cheeseburgers without any cost increase beyond the cost of the first cheeseburger.

Strand            **Algebra and  
Functions (AF)**

Standard        **AF4.1**  
**Solve two-step linear equations  
and inequalities in one variable  
over the rational numbers,  
interpret the solution or solutions  
in the context from which they  
arose, and verify the  
reasonableness of the results.**

Constructs      **Procedural Skills,  
Conceptual  
Understanding,  
Problem Solving**

**In the inequality  $2x + \$10,000 \geq \$70,000$ ,  
 $x$  represents the salary of an employee in a  
school district. Which phrase most  
accurately describes the employee's salary?**

- A**    At least \$30,000
- B**    At most \$30,000
- C**    Less than \$30,000
- D**    More than \$30,000

CAHSEE test questions for this standard may focus on any of its components, including solving two-step linear equations, solving two-step inequalities, interpreting the solutions of equations or inequalities, and judging the reasonableness of the solutions or equations or inequalities.

### *Sample Test Question*

The sample test question presents an inequality that represents an employee's salary and asks students to interpret the solution of the inequality in terms of this context. The correct answer is choice A. Students should recognize that the inequality has the solution  $x \geq \$30,000$  and that the correct way to state this inequality is "at least \$30,000."

### *Analysis of Distractors*

The distractors offer incorrect solutions for the inequality and/or ways to express the mathematical notation. Distractor B represents  $x \leq \$30,000$ , which is stated as "at most \$30,000." Distractor C represents  $x < \$30,000$ , or "less than \$30,000." Distractor D represents  $x > \$30,000$ , or "more than \$30,000."

Strand            **Algebra and Functions (AF)**

Standard        **AF4.2**  
**Solve multi-step problems involving rate, average speed, distance, and time or a direct variation.**

Constructs      **Procedural Skills, Conceptual Understanding, Problem Solving**

**The diameter of a tree trunk varies directly with the age of the tree. A 45-year-old tree has a trunk diameter of 18 inches. What is the age of a tree that has a trunk diameter of 20 inches?**

- A**    47 years
- B**    50 years
- C**    63 years
- D**    90 years

Problem solving is a significant higher-order thinking skill that enables students to apply their mathematical knowledge to real-world situations. CAHSEE test questions for this standard may require students either to solve a specific problem or to determine the equation that should be used to solve the problem. Test questions may also require students to understand the concept of direct variation and to recognize that direct variation may also be expressed as a linear function. This standard is closely related to Algebra 1 standard 5.0, which requires students to solve a variety of problems such as rate, work, and percent mixture using algebraic methods.

### *Sample Test Question*

The sample question provides a ratio between the age of a tree and the diameter of its trunk and asks students to determine the age of a second tree, given its diameter. The correct answer is choice B. Students may choose to use a variety of appropriate solutions that rely on the correspondence between the age of the tree and its diameter. One method is to develop the equation representing the direct variation between the ages of the trees and their corresponding diameters:  $\frac{45}{18} = \frac{x}{20}$  and solve for  $x$ .

### *Analysis of Distractors*

The distractors offer solutions that use the values of the problem incorrectly and thus reflect a misunderstanding of the concept of direct variation. Distractor A represents the error of adding the value of the difference between the two diameters ( $20 - 18 = 2$ ) to the age of the first tree in order to obtain the age of the second tree. Distractor C represents the addition of the two values associated with the first tree ( $45 + 18 = 63$ ). Distractor D represents the product of the age of the first tree (45) and the difference between the two diameters (2).

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## The Measurement and Geometry Strand

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As students relate their experiences from earlier classroom instruction in measurement and geometry to situations in their everyday lives, their knowledge and ability to apply this knowledge increase in depth and sophistication. To demonstrate understanding in this CAHSEE strand, students must be able to select and use appropriate units, estimate and calculate measurements for the length, area, and volume of geometric figures, understand scaling in scale drawings and how changes in linear dimension affect area and volume, and solve problems involving dimensional analysis and conversion from one unit to another.

To demonstrate achievement of the knowledge and skills in the measurement component of the strand, students should be able to use both metric and customary units of measurement for the following:

- determining the relationship between different units within the same system and converting from one unit to another within and between measurement systems
- using scale drawings and models to determine measurements of the original
- solving problems involving dimensional analysis for rates and other compound units
- relating the effect of changing the choice of a linear unit on the related square and cubic units for area and volume, respectively

The geometry component of this strand includes computing the perimeter, area, and volume of the most common 2- and 3-dimensional figures, using these common figures to estimate or compute the area of more complex objects.

To demonstrate acquisition of the knowledge and skills in the geometry component of the strand, students should be able to:

- describe, classify, and understand relationships between length, area, and volume among types of 2- and 3-dimensional objects
- use coordinate geometry to represent and examine the properties of figures and their images under translation and reflection
- understand and use the Pythagorean theorem
- recognize and demonstrate understanding of congruence in terms of the sides and angles of 2-dimensional figures

When CAHSEE items require students to use formulas, the formulas are provided within parentheses in the stem. All formulas the students may use, including the estimated value of  $\pi$ , will be provided *except* for the following:

- Perimeter of a polygon and the circumference of a circle
- Area of a triangle or parallelogram (including rectangles)
- Volume of a rectangular prism

The formula for finding the area of a nontraditional figure such as a rhombus will be provided.

The 10 California Content Standards covered by the CAHSEE Measurement and Geometry strand are discussed in the following pages.



Strand	<b>Measurement and Geometry (MG)</b>	<b>Juanita exercised for one hour. How many seconds did Juanita exercise?</b>  <b>A</b> 60 <b>B</b> 120 <b>C</b> 360 <b>D</b> 3,600
Standard	<b>MG1.1</b>	
	<b>Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).</b>	
Constructs	<b>Procedural Skills, Conceptual Understanding</b>	

Comparing units and computing the effect of changing units are essential skills for acquiring knowledge in mathematics and science disciplines. CAHSEE test questions for this standard require students to convert between two units of measurement within the same system or between two different systems of measurement. Conversion formulas are provided for test questions that require the student to convert between less commonly utilized units (such as pints to gallons), systems of measurement, square units, and cubic units.

### *Sample Test Question*

The sample question requires students to convert common units of time. The correct answer is choice D. Students must compute the number of seconds in 1 hour by multiplying the 60 seconds in 1 minute by the 60 minutes in 1 hour, or  $60 \times 60$ , or 3600 seconds.

### *Analysis of Distractors*

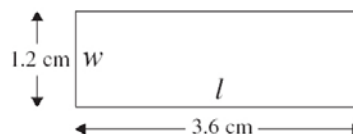
The distractors represent errors in performing the conversions. Distractor A gives the number of seconds in 1 minute or the number of minutes in 1 hour and thus represents the failure to perform the second step of the problem. Distractor B represents adding, rather than multiplying, the number of seconds in a minute and the number of minutes in an hour. Distractor C represents a computation error in multiplying 60 by 60.

Strand **Measurement and Geometry (MG)**

Standard **MG1.2**  
**Construct and read drawings and models made to scale.**

Constructs **Conceptual Understanding, Problem Solving**

The actual width ( $w$ ) of a rectangle is **18 centimeters (cm)**. Use the scale drawing of the rectangle to find the actual length ( $l$ ).



- A 6 cm
- B 24 cm
- C 36 cm
- D 54 cm

The ability to move back and forth between a scale drawing or model and a real object is essential for understanding representation and well as proportional reasoning. Test questions on the CAHSEE focus on the second component of this standard, reading scale drawings and models. Students will be asked to read and interpret drawings and models made to scale. Students may also be asked to apply given measurements to determine the scale of a figure.

#### *Sample Test Question*

The sample test question presents a scale drawing of a rectangle with width 1.2 cm and length 3.6 cm and gives the actual value of the width as 18 cm. The correct answer is choice D. Students must recognize that the relationship between the width of the scale drawing and the actual width of the rectangle is the same as that between the length of the scale drawing and the actual length. Students may use a variety of approaches to solve the problem, including setting up a proportion similar to  $\frac{1.2}{18} = \frac{3.6}{l}$  and solving for  $l$ .

#### *Analysis of Distractors*

The distractors represent errors in using the values presented in the stem. Distractor A results from use of an incorrect relationship between the parts of the proportion:  $(18 \div 3.6) \times 1.2 = 6$ . Distractor B results from subtracting the width from the length and multiplying by 10. Distractor C results from using 3.6 as twice, rather than three times, 1.2 and then multiplying by 2.

Strand	<b>Measurement and Geometry (MG)</b>	<b>Sixty miles per hour is the same rate as which of the following?</b>  <b>A</b> 1 mile per minute <b>B</b> 1 mile per second <b>C</b> 6 miles per minute <b>D</b> 360 miles per second
Standard	<b>MG1.3</b> <b>Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.</b>	
Constructs	<b>Procedural Skills, Conceptual Understanding, Problem Solving</b>	

Dimensional analysis is a critical skill in physical science, engineering, and the social sciences. Students should be familiar with the rates named in the standard as well as other commonly used measures (e.g., kilowatt hours, foot-pounds, acre-feet). Test questions on the CAHSEE for this standard focus on all three of its components: measures as rates, measures as products, and reasonableness of results.

### *Sample Test Question*

The sample question requires the student to find an equivalent rate to 60 miles per hour. The correct answer is choice A. Students may reason that another way to express the given rate is 60 miles per 60 minutes, since 1 hour is equivalent to 60 minutes. The expression is  $60 \text{ miles/hour} = 60 \text{ miles}/60 \text{ minutes}$  and that since  $60 \div 60 = 1$ , the rate is equivalent to 1 mile/minute. Students should also be encouraged to evaluate the answer choices in the question for their reasonableness as a restatement of this real-world rate.

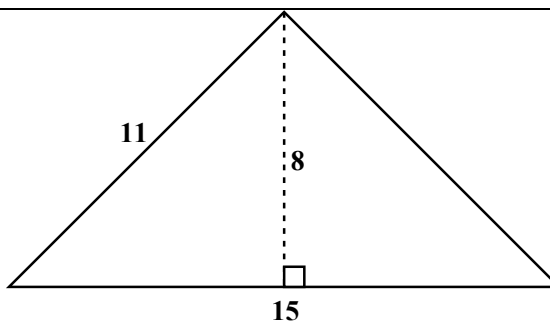
### *Analysis of Distractors*

The distractors represent misunderstandings of the required calculations. Distractor B results from incorrectly equating 1 hour to 60 seconds. Distractor C is obtained by dividing 60 miles per hour by 10, rather than by 60. Distractor D is obtained by multiplying 60 by 6.

Strand **Measurement and Geometry (MG)**

Standard **MG2.1**  
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Constructs **Procedural Skills, Conceptual Understanding**



**What is the area of the triangle shown above?**

- A 44 square units
- B 60 square units
- C 88 square units
- D 120 square units

All students should know how to compute the area and volume of basic figures and also how to apply basic formulas in many areas to solve problems. This standard requires students to find the perimeter and area of 2-dimensional figures and the surface area and volume of 3-dimensional figures. The figures tested on the CAHSEE include parallelograms, trapezoids, triangles, circles, prisms, and cylinders. Students are required to know the following formulas:

- perimeter of a polygon
- circumference of a circle
- area of a triangle
- area of a parallelogram (including rectangles)
- volume of a rectangular prism

These formulas will not be given with the test questions. The estimated value of  $\pi$  will be given unless  $\pi$  is included in the answer choices.

### *Sample Test Question*

The sample question presents a triangle with the dimensions labeled and asks students to compute the area. The correct answer is choice B. Students should know the formula

$\frac{1}{2} \times bh$  and use it to determine that since the height of the triangle is 8 units and the base is 15 units, the area is 60 square units.

*Analysis of Distractors*

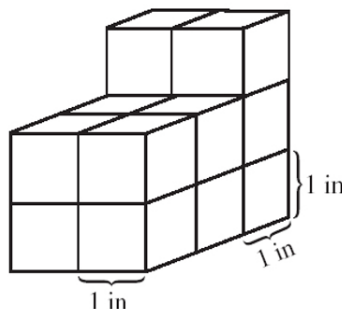
The distractors represent misapplications of the area formula. Distractor A uses 11 as the base, rather than 15. Distractor C uses 11 as the base and also fails to take  $\frac{1}{2}$  of the product of the base and height. Distractor D applies the formula to the correct dimensions but contains the common error of failing to take  $\frac{1}{2}$  of the product of the base and height.

Strand **Measurement and Geometry (MG)**

Standard **MG2.2**  
**Estimate and compute the [surface] area of more complex or irregular two-and three-dimensional figures by breaking the figures down into more basic geometric objects.**

Constructs **Procedural Skills, Conceptual Understanding**

One-inch cubes are stacked as shown in the drawing below.



What is the total surface area?

- A 19 in<sup>2</sup>
- B 29 in<sup>2</sup>
- C 32 in<sup>2</sup>
- D 38 in<sup>2</sup>

This standard emphasizes the development of problem-solving skills with visual tools. CAHSEE test questions for this standard focus on both of its components. The first component requires students to identify the measurable and/or computable parts of a shape or structure. This identification task is largely dependent on the ability to visualize the familiar geometric structures that make up a more complex figure. Where estimation is required, students may be given dimensions of known shapes, scale and proportion, or grids in the visual prompt. The second component requires students to estimate and/or compute the area of the subdivided portions of a figure. Frequently the dimensions of the component parts are not given directly by labeled measurements but must be determined by such means as adding or subtracting lengths or extending lines.

Formulas will be provided where necessary, *except* for the following:

- perimeter of a polygon
- circumference of a circle
- area of a triangle
- area of a parallelogram (including rectangles)
- volume of a rectangular prism

In estimation problems, the approximate value of  $\pi$  that should be used to obtain the correct answer will be provided as part of the stem. Otherwise, the answer will include  $\pi$  (e.g.,  $12 + 3\pi$ ).

*Sample Test Question*

The sample question presents a drawing of an assembly of 1-inch cubes and asks students to determine the surface area. The correct answer is choice D. The foreground portion of the object contains a 2 by 2 by 2-cube grouping, and the background contains a 1 by 3-cube grouping. To find the surface area, students must visually assemble the surface of the individual cubes that make up the entire object and recognize that the object has 8 planar faces: right, rear, left, front, bottom, front top, rear face, and rear top. The surface areas of each corresponding face are 7, 6, 7, 4, 6, 4, 2, and 2 square inches, and the sum of the areas of these faces is 38 square inches.

*Analysis of Distractors*

The distractors represent inaccurate visualizations of the faces of the object. Distractor A includes only half of the exterior faces. Distractor B fails to include either the left or right face and either the rear face or rear top. Distractor C fails to include the bottom or rear of the figure.

**Strand Measurement and Geometry (MG)**

**Standard MG2.3**  
**Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.**

**Constructs Procedural Skills, Conceptual Understanding, Problem Solving**

A cereal manufacturer needs a box that can hold twice as much cereal as the box shown below.



Which of the following changes will result in the desired box? ( $V = lwh$ )

- A Double the height only.
- B Double both the length and width.
- C Double both the length and height.
- D Double the length, width and height.

Scaling effects provide insights for learning advanced topics in all disciplines, and this content standard requires students to understand how changes in dimensions affect the size of objects. The standard contains two components, one addressing the effect of the scale factor on area, and the other addressing the effect of the scale factor on volume. Students should understand that when the dimensions of a two-dimensional object are multiplied by a scale factor, the resulting area is multiplied by the square of the scale factor. Students should also understand that when the dimensions of a three-dimensional object are multiplied by a scale factor, the resulting volume is multiplied by the cube of the scale factor.

*Sample Test Question*

The sample question presents a rectangular prism and asks students to determine the change that will double the volume of the prism. The correct answer is choice A. Students should recognize that doubling one dimension will be sufficient to double the original volume.

*Analysis of Distractors*

The distractors represent conceptual misunderstandings of the effect of the change of dimensions on volume. Students who choose distractors B or C may not recognize that doubling two of the dimensions will result in a new volume that is four times the original volume. Students who choose distractor D may not understand that doubling all three dimensions will result in a new volume that is eight times the original volume.



Strand	<b>Measurement and Geometry (MG)</b>	<p><b>One cubic inch is approximately equal to 16.38 cubic centimeters. Approximately how many cubic centimeters are there in 3 cubic inches?</b></p> <p>A 5.46</p> <p>B 13.38</p> <p>C 19.38</p> <p>D 49.14</p>
Standard	<b>MG2.4</b>	
Construct	<b>Conceptual Understanding</b>	
<p><b>Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or <math>[1 \text{ ft}^2] = [144 \text{ in}^2]</math>, 1 cubic inch is approximately 16.38 cubic centimeters or <math>[1 \text{ in}^3] = [16.38 \text{ cm}^3]</math>).</b></p>		

Unit conversions for area and volume have practical applications in students' lives. CAHSEE test questions for this standard require students to relate the changes in measurement with a change of scale to the units used and to convert between units. The emphasis is on the relationship between linear units for distance, square units for area, and cubic units for volume. Conversion formulas will be provided for linear measures except for inches to feet to yards or conversions within the metric system.

### *Sample Test Question*

The sample question gives students an equivalence between inches and centimeters and asks for a computation based on that equivalence. The correct answer is choice D. Students should perform the computation ( $16.38 \times 3 = 49.14$ ) or use estimation to determine the correct answer ( $3 \times 18 = 48$ , approximating 49.14).

### *Analysis of Distractors*

The distractors represent incorrect choices of mathematical operations. Distractor A is obtained by dividing 16.38 by 3. Distractor B is obtained by subtracting 3 from 16.38. Distractor C is obtained by adding 3 to 16.38.

Strand

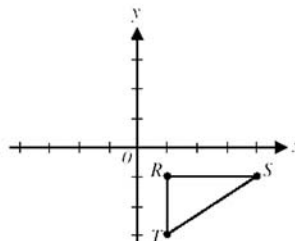
**Measurement  
and Geometry  
(MG)**

Standard **MG3.2**

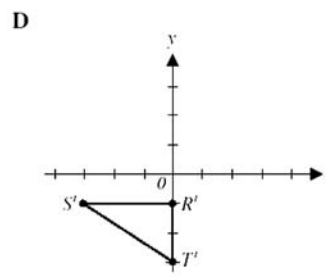
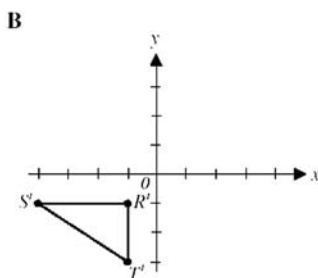
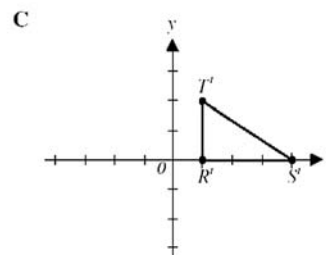
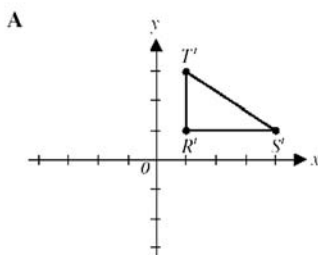
**Understand and use  
coordinate graphs to  
plot simple figures,  
determine lengths and  
areas related to them,  
and determine their  
image under translations  
and reflections.**

Constructs

**Procedural Skills,  
Conceptual  
Understanding,  
Problem Solving**



Which of the following triangles  $R'S'T'$  is the image of triangle  $RST$  that results from reflecting triangle  $RST$  across the  $y$ -axis?



CAHSEE test questions for this standard assess students' understanding of all components of the standard: plotting with ordered pairs, determining lengths and areas from plotted figures, and finding images following transformations by translations and reflections.

### *Sample Test Question*

The sample question requires students to choose the correct translation of an object (triangle  $RST$ ) by reflecting it across the  $y$ -axis. The correct answer is choice B. Students must recognize the reflective correspondence between points  $R$  and  $R'$ ,  $S$  and  $S'$ , and  $T$  and  $T'$ , and that triangle  $R'S'T'$  is the reflective image, across the  $y$ -axis, of triangle  $RST$ .

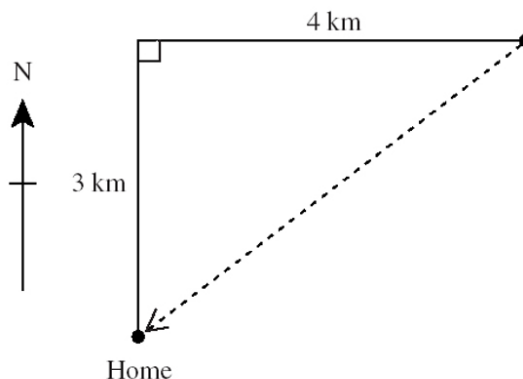
### *Analysis of Distractors*

Distractor A is the reflection of triangle  $RST$  across the  $x$ -axis. Distractor C is the reflection of triangle  $RST$  across the  $x$ -axis and its translation by one unit down. Distractor D is the reflection of triangle  $RST$  across the  $y$ -axis and its translation by 1 unit to the right.

Strand **Measurement and Geometry (MG)**

Standard **MG3.3**  
**Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.**

Constructs **Conceptual Understanding, Problem Solving**



The club members hiked 3 kilometers north and 4 kilometers east, but then went directly home as shown by the dotted line. How far did they travel to get home?

- A 4 km
- B 5 km
- C 6 km
- D 7 km

The Pythagorean theorem is important for its problem-solving function as well as its role as a bridge between geometry and algebra. CAHSEE test questions assessing this standard address the following components of the standard: using the Pythagorean theorem to find the missing base, altitude, or hypotenuse length in a right triangle and using the Pythagorean theorem to find lengths of line segments in figures other than triangles.

### *Sample Test Question*

The sample question presents a right triangle with base 3 km, altitude 4 km, and an unlabeled hypotenuse, in the context of a hiking trip. The correct answer is choice B. Students should understand that use of the Pythagorean theorem is appropriate to find how far the club members hiked, which is the length of the hypotenuse ( $3^2 + 4^2 = c^2$ ). Students may also be expected to have memorized common right triangles such as this one.

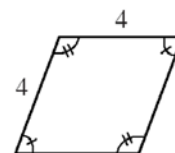
### *Analysis of Distractors*

Distractor A is the same length as the altitude. Distractor C is an incorrect solution to the equation that applies the Pythagorean theorem. Distractor D is the sum of the base and altitude.

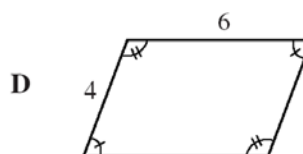
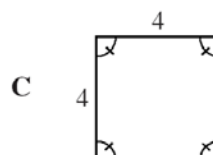
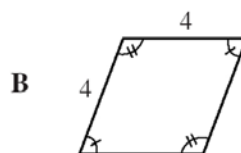
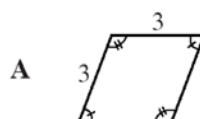
Strand **Measurement and Geometry (MG)**

Standard **MG3.4**  
**Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about relationships between the sides and angles of the two figures.**

Construct **Conceptual Understanding**



Which figure is congruent to the figure shown above?



CAHSEE test questions for this standard require students to discriminate between figures that are or are not congruent to a given figure. Students may also be asked to identify specific reasons or other supporting information that would determine congruence between two figures. Relationships between sides and angles of figures are significant because they may indicate the presence or absence of congruence by axiomatic reasoning, such as side-angle-side congruence. Items in this standard will not involve the use of acronyms for side and angle relationships of congruent figures (e.g. SAS, SSS).

*Sample Test Question*

The sample question presents a figure with the dimensions of the sides given and the angles marked. Students are asked to use the characteristics of this figure to find the congruent figure among the answer choices. The correct answer is choice B. Students should understand that two polygons are congruent if all corresponding sides and angles are congruent. Reasoning from this basis, students should recognize that the figure in choice B is congruent to the given figure because of the correspondence with sides of length 4 and angles with two marks. Students should also understand that because both figures are rhombi with non-congruent adjacent angles, all four figures are congruent and the opposite angles are congruent.

*Analysis of Distractors*

Distractor A provides a similar, but not congruent, figure. Distractor C is a square, and distractor D is a parallelogram with non-congruent adjacent sides.

## The Algebra 1 Strand

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The Algebra 1 strand builds upon students' knowledge and skills developed from their experience with linear functions, tables, graphs, verbal rules, and symbolic rules. As students deepen their understanding of relations and functions, they will expand their capacity to make meaningful use of new types of functions, including polynomial, exponential, rational, and periodic functions. Important new learning includes combining functions, expressing functions in equivalent forms, and finding inverses. This experience leads to more global understanding of classes of functions as a concept and the recognition of the significant characteristics of various classes.

To demonstrate achievement in the Algebra 1 strand, students must also develop insights into mathematical abstraction and structure. Students should develop an understanding of the algebraic properties that govern the manipulation of symbols in expressions. As students become more familiar with these types of abstractions, they develop the means to solve equations and inequalities, express equivalent forms, and assert proofs.

Facility with abstraction and deeper knowledge of functions and relations give students more powerful mathematical tools to analyze and describe situations. Tools such as graphs and other visual representations of phenomena provide additional insights into problems and applications.

Standards in the Algebra 1 strand include performing operations such as opposite (additive inverse), reciprocal, and root; solving equations and inequalities with absolute values; simplifying expressions; solving multi-step problems with linear equations and inequalities; graphing linear equations and finding the  $x$  and  $y$ -intercepts; verifying points on a line given an equation; deriving linear equations; understanding and using the relationship between parallel lines and slopes; solving systems of linear equations, including meaningfully interpreting their graphical representations; performing operations and solving multi-step problems with monomials and polynomials; and solving rate, work, and percent mixture problems.

The 10 specific California Content Standards covered by the CAHSEE Algebra 1 strand are discussed in the following pages.

Strand                      **Algebra 1 (AI)**

Standard                **AI2.0**

**Students understand and use such operations as taking the opposite, finding the reciprocal, and taking a root, and raising to a fractional power. They understand and use the rules of exponents.\***

Construct                **Conceptual  
Understanding**

**The perimeter,  $P$ , of a square may be found by using the formula  $(\frac{1}{4})P = \sqrt{A}$ , where  $A$  is the area of the square. What is the perimeter of the square with an area of 36 square inches?**

- A     9 inches**
- B     12 inches**
- C     24 inches**
- D     72 inches**

Facility with inverse operations is critical for students as they solve equations and inequalities. CAHSEE test questions for this content standard focus on two of the stated components: finding the opposite (additive inverse) and finding the reciprocal. As students gain facility in algebraic reasoning, they should recognize the usefulness of finding the additive inverse and reciprocal in simplifying equations and inequalities.

### *Sample Test Question*

The sample question gives students a formula for finding the perimeter of a square using the reciprocal  $\frac{1}{4}P$ . The correct answer is choice C. To solve the equation for  $P$ , students should substitute 36 for  $A$  and calculate that  $P = 4\sqrt{36}$ , or 24.

### *Analysis of Distractors*

The distractors represent misapplications of the formula. Distractor A is obtained by dividing 36 by 4. Distractor B is obtained by taking the square root of 36 but multiplying by 2 instead of 4. Distractor D is obtained by multiplying  $36 \times 2$ .

\* The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Strand            **Algebra 1 (AI)**

Standard        **AI3.0**  
**Students solve equations and inequalities involving absolute values.**

Constructs      **Procedural Skills, Conceptual Understanding, Problem Solving**

**If  $x$  is an integer, what is the solution to  $|x - 3| < 1$ ?**

- A**     $\{-3\}$
- B**     $\{-3, -2, -1, 0, 1\}$
- C**     $\{3\}$
- D**     $\{-1, 0, 1, 2, 3\}$

For success in algebra, students should understand the concept of absolute value and the ways its meaning is used in solving equations and inequalities. CAHSEE test questions for this standard focus on both of its components: solving equations involving absolute values and solving inequalities involving absolute values. On the exam, the tested inequalities will involve only integers and will not use nested sets of inequalities.

### *Sample Test Question*

In the sample question, students are asked to solve an inequality involving absolute value. The correct answer is choice C. Students should recognize that this inequality,  $|x - 3| < 1$ , is equivalent to  $-1 < x - 3 < 1$ . By adding 3 to each member of the inequality, the following equivalence is obtained:  $2 < x < 4$ . Since  $x$  is an integer, the solution is 3 because 3 is the only integer greater than 2 and less than 4.

### *Analysis of Distractors*

Distractor A is the negative of the correct answer and results from a misunderstanding of absolute value. Distractors B and D result from setting up the inequality improperly and then adding  $-3$  to only two members of the inequality.



Strand            **Algebra 1 (AI)**

Standard        **AI4.0**

**Students simplify expressions before solving linear equations and inequalities in one variable, such as  $3(2x - 5) + 4(x - 2) = 12$ .**

Constructs      **Conceptual Understanding, Problem Solving**

**Which of the following is equivalent to  $4(x + 5) - 3(x + 2) = 14$ ?**

**A**     $4x + 20 - 3x - 6 = 14$

**B**     $4x + 5 - 3x + 6 = 14$

**C**     $4x + 5 - 3x + 2 = 14$

**D**     $4x + 20 - 3x - 2 = 14$

Finding ways to simplify expressions before trying to solve an equation or inequality is a valuable problem-solving skill. CAHSEE test questions for this content standard involve both equations and inequalities and focus on simplification rather than solution.

### *Sample Test Question*

In the sample question, students are asked to identify an equation that is equivalent to the given equation, thereby determining the first step in simplification. The correct answer is choice A. Students should understand that for the given equation the first step in simplifying is to expand the quantities in parentheses with each coefficient:

$$4x + 20 - 3x - 6 = 14.$$

### *Analysis of Distractors*

The distractors present incorrect simplifications. Distractor B presents the failure to multiply 4 by 5 in the first set of parentheses and to obtain a product of +6 from multiplying  $-3$  by 2. Distractor C presents the failure to multiply 4 by 5 in the first set of parentheses and  $-3$  by 2 in the second set of parentheses. Distractor D presents the failure to multiply  $-3$  by 2 in the second set of parentheses.

Strand                      **Algebra 1 (AI)**

Standard                **AI5.0**

**Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.**

Constructs            **Procedural Skills, Conceptual Understanding, Problem Solving**

**Solve for  $x$ .**

$$5(2x - 3) - 6x < 9$$

- A**     $x < -1.5$
- B**     $x < 1.5$
- C**     $x < 3$
- D**     $x < 6$

Solving multi-step problems and word problems can help students develop problem-solving skills and enhance their ability to think algebraically. CAHSEE test questions for this standard focus on both of its components: finding solutions to linear equations and inequalities and providing justification for each step in the solution. Justification requires the identification of appropriate specific steps in the solution process that contribute to solving the equation or inequality.

### *Sample Test Question*

The sample question presents an inequality and asks students to solve for the variable. The correct answer is choice D. Students should recognize that the inequality requires expanding the quantity in parentheses by multiplying it by the coefficient, combining like terms, adding 15 to both sides of the equation, and dividing both sides by 4 to obtain the value for  $x$ .

### *Analysis of Distractors*

The distractors present mistakes in the solution process. Distractor A is obtained by subtracting 15 in the third step. Distractor B is obtained by subtracting 15 in the third step and dividing by  $-4$  in the fourth step, or finding  $+6$  as the sum of  $15 + 9$ . Distractor C is obtained by finding  $8x$  as the sum of  $2x$  and  $-6x$ , after failing to multiply 5 by 2 in the first set of parentheses.

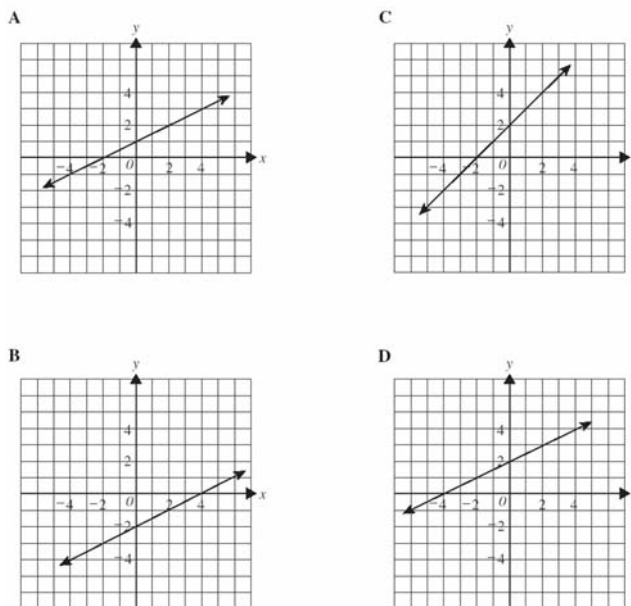
Strand **Algebra 1 (AI)**

Standard **AI6.0**

**Students graph a linear equation and compute the  $x$ - and  $y$ -intercepts (e.g., graph  $2x + 6y = 4$ ). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by  $2x + 6y < 4$ ).\***

Constructs **Procedural Skills, Conceptual Understanding**

Which of the following is the graph of  $y = \frac{1}{2}x + 2$ ?



CAHSEE test questions for this standard focus on two components: graphing the linear equation and finding the  $x$ - and  $y$ -intercepts. Students may be asked to identify the graph that corresponds to a given equation or to identify the equation that corresponds to a given graph. The  $x$ - and  $y$ -intercepts may be identified with a single number or with ordered pairs.

*Sample Test Question*

The sample question presents an equation and asks students to identify the correct graph of the equation from among four choices. The correct answer is choice D. Students

should recognize that the equation,  $y = \frac{1}{2}x + 2$ , is in the form  $y = mx + b$  and use this

information to determine the correct graph: The variable  $b$  represents the  $y$ -intercept, which in this case is 2, and the variable  $m$  represents the slope of the graph, which in this case is  $\frac{1}{2}$ .

*Analysis of Distractors*

Distractor A displays a line with a slope of  $\frac{1}{2}$  and  $y$ -intercept of 1. Distractor B displays

a line with a slope of  $\frac{1}{2}$  and  $y$ -intercept of -2. Distractor C displays a line with a slope of 1 and  $y$ -intercept of 2.

\* The CAHSEE test blueprint does not include the crossed-out portion of this content standard because the test questions are all multiple choice.

Strand	<b>Algebra 1 (AI)</b>	<b>Which of the following points lies on the line <math>4x + 5y = 20</math>?</b>  <b>A</b> (0, 4) <b>B</b> (0, 5) <b>C</b> (4, 5) <b>D</b> (5, 4)
Standard	<b>AI7.0</b>	
<b>Students verify that a point lies on a line, given an equation of the line. Students are able to derive - linear equations by using the point slope formula.*</b>		
Constructs	<b>Procedural Skills, Conceptual Understanding, Problem Solving</b>	

CAHSEE test questions for this standard require students to select a set of one or more points, either by ordered pairs or by graphical location, that lie on the graph of a given linear equation, or to select an equation, either by its algebraic notation or by its graph, whose graph includes one or more specified points. Components of the standard include verifying that a point lies on a given line and deriving an equation from information given about the line. To verify that points do or do not lie on a given line, students may use substitution of  $x$  or  $y$  values to find corresponding ordered pairs.

### *Sample Test Question*

The sample question presents an equation and asks students to identify an ordered pair that would lie on the graph of that equation. The correct answer is choice A. Students should recognize that the equation  $4x + 5y = 20$  represents true statements for certain real number values of  $x$  and  $y$ . The values may be tested by substituting them for  $x$  and  $y$ , respectively, in the equation. If  $x = 0$  and  $y = 4$ , then  $4(0) + 5(4) = 20$  is an equation.

### *Analysis of Distractors*

Distractor B would mean substituting 0 for  $x$  and 5 for  $y$ , the result being that  $4(0) + 5(5) = 25$  rather than 20. Distractor C means substituting 4 for  $x$  and 5 for  $y$ , the result being that  $4(4) + 5(5) = 41$  rather than 20. Distractor D means substituting 5 for  $x$  and 4 for  $y$ , the result being that  $4(5) + 5(4) = 40$  rather than 20.

\* The CAHSEE test blueprint does not include the crossed-out portion of this content standard because the test questions are all multiple choice.

Strand            **Algebra 1 (AI)**

 Standard        **AI8.0**

**Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. ~~Students are able to find the equation of a line perpendicular to a given line that passes through a given point.\*~~**

Constructs       **Conceptual Understanding, Problem Solving**

What is the slope of a line parallel to the line

$$y = \frac{1}{3}x + 2?$$

 A      $-3$ 

 B      $-\frac{1}{3}$ 

 C      $\frac{1}{3}$ 

 D      $2$ 

To demonstrate understanding of this content standard, students must know that parallel lines have equivalent slopes and different  $x$ - and  $y$ -intercepts. CAHSEE test questions for this standard may require students to find the slope of a line parallel to a given line, to identify pairs of parallel lines from their slopes, or to identify lines not parallel to a given line from a given or derived slope.

### *Sample Test Question*

The sample question asks students to determine the slope of a line parallel to a given line. The correct answer is choice C. Students should know that parallel lines have equivalent slopes. They must also recognize that the slope of the line that is represented by the equation  $y = \frac{1}{3}x + 2$  is  $\frac{1}{3}$ .

### *Analysis of Distractors*

Distractor A is the negative reciprocal of the slope. Distractor B is the additive inverse of the slope. Distractor D is the  $y$ -intercept (constant).

\* The CAHSEE test blueprint does not include the crossed-out portion of this content standard because the test questions are all multiple choice.

Strand            **Algebra 1 (A1)**

Standard        **AI9.0**

**Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.**

Constructs      **Procedural Skills,  
Conceptual  
Understanding,  
Problem Solving**

$$\begin{cases} 7x + 3y = -8 \\ -4x - y = 6 \end{cases}$$

**What is the solution to the system of equations shown above?**

- A**     $(-2, -2)$
- B**     $(-2, 2)$
- C**     $(2, -2)$
- D**     $(2, 2)$

Many real-world situations are most appropriately modeled as systems of equations, and graphs of these kinds of systems are common in newspapers and other media. CAHSEE test questions for this content standard focus on four components: solving systems of linear equations, interpreting the solutions graphically, solving a system of linear inequalities, and determining the solution sets.

### *Sample Test Question*

The sample question presents a system of equations and asks student to find the solution. The correct answer is choice B. To solve this sample problem, students should use a method such as the following:

(multiply the second equation by 3)	$-12x - 3y = 18$
(add to the first equation )	$-5x = 10$
(divide by 5)	$x = -2$
(substitute $x = -2$ in the first equation )	$7(-2) + 3y = -8$
(expand)	$-14 + 3y = -8$
(combine)	$3y = 6$
(divide by 3)	$y = 2$

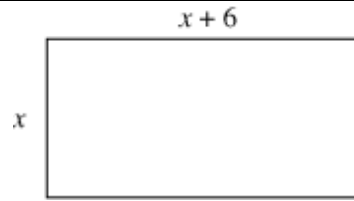
### *Analysis of Distractors*

Distractor A results from errors in substitution and/or computation, as does distractor D. Distractor C presents a misunderstanding of the correct order of the  $x$  and  $y$  values.

Strand **Algebra 1 (AI)**

Standard **AI10.0**  
**Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.**

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



**The length of the rectangle above is 6 units longer than the width. Which expression could be used to represent the area of the rectangle?**

- A  $x^2 + 6x$
- B  $x^2 - 36$
- C  $x^2 + 6x + 6$
- D  $x^2 + 12x + 36$

This standard requires students to accurately execute arithmetic operations on monomials and polynomials and to select and use these techniques to solve problems.

*Sample Test Question*

The sample question presents a rectangle with length and width labeled with a monomial and a binomial, and students are asked to determine the expression that would correctly represent the area. The correct answer is choice A. Students must know to multiply the length by the width in order to find the area and then perform the computation correctly.

*Analysis of Distractors*

Distractors B, C, and D represent examples of incorrect multiplication of the monomial by the binomial.

Strand            **Algebra 1 (AI)**

Standard        **AI15.0**  
**Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.**

Constructs      **Procedural Skills, Conceptual Understanding, Problem Solving**

**Mr. Jacobs can correct 150 quizzes in 50 minutes. His student aide can correct 150 quizzes in 75 minutes. Working together, how many minutes will it take them to correct 150 quizzes?**

- A     30**
- B     60**
- C     63**
- D    125**

The key assessment of student learning in any subject is whether or not students can apply their knowledge and skills to a new problem situation. CAHSEE test questions written for this standard focus on students' ability to apply their mathematical skills and knowledge to solve rate problems, work problems, and percent mixture problems.

#### *Sample Test Question*

The sample question presents a rate problem in the context of grading quizzes. The correct answer is choice A. Students should recognize that correcting 150 quizzes in 50 minutes is equivalent to the rate of 3 quizzes per minute and that correcting 150 quizzes in 75 minutes is equivalent to the rate of 2 quizzes per minute. Working together, Mr. Jacobs and his aide can correct 5 quizzes each minute:

$$150 \text{ quizzes} \div 5 \text{ quizzes/minute} = 30 \text{ minutes.}$$

#### *Analysis of Distractors*

The distractors represent misunderstandings of a rate problem. Distractor B represents the number of minutes it would take both individuals to complete the task at the rate of 2.5 quizzes/minute—the average of their rate. Distractor C is the approximate average of their time for 150 quizzes. Distractor D is the sum of the number of minutes each takes to correct 150 quizzes (50 + 75).



## **The Mathematical Reasoning Strand**

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Reasoning is an integral part of mathematics and requires several important skills, including examining patterns for regularities, making and testing conjectures about generalizations, and using formal inductive and deductive reasoning to formulate mathematical arguments. Mastery of each of these elements of reasoning requires students to work with diverse problems and activities.

Activities in this strand require language with sufficient precision, clarity, and appropriateness to support rigorous thinking. Standards in mathematical reasoning require students to analyze problems by identifying relationships, to formulate and justify conjectures, to use estimation on the basis of numerical or graphical information, to use inductive and deductive reasoning, to evaluate the reasonableness of solutions, and to generalize results and apply them to new problems.

Each question in this standard also is classified within one of the other five mathematical strands for purposes of reporting student scores.

The seven specific California Content Standards covered by the CAHSEE Mathematical Reasoning strand are discussed in the following pages.

Strand            **Mathematical Reasoning (MR)**

Standard        **MR1.1**  
**Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.**

Constructs       **Procedural Skills, Conceptual Understanding, Problem Solving**

**Chris drove 100 kilometers from San Francisco to Santa Cruz in 2 hours and 30 minutes. What computation will give Chris' average speed, in kilometers per hour?**

- A    Divide 100 by 2.5.
- B    Divide 100 by 2.3.
- C    Multiply 100 by 2.5.
- D    Multiply 100 by 2.3.

Students must be able to analyze situations to clarify a problem and to identify those elements that will make it possible to solve the problem. CAHSEE test questions for this standard emphasize the analysis of problems rather than their solutions. The components of the standard include the following: determining relationships, discriminating between relevant and irrelevant information, identifying missing information, sequencing and prioritizing information, and observing and identifying algebraic and geometric patterns.

### *Sample Test Question*

The sample question requires students to understand the relationship between the given distance (100 km), the given time (2 hours 30 minutes), and the rate (unknown). The correct answer is choice A. Students must recognize that rate, Chris' average speed, is the unknown variable and must know how to use the distance, rate, time equation to determine the rate. This question is also classified in the Algebra and Functions strand for purposes of reporting student scores.

### *Analysis of Distractors*

The distractors represent errors in the application of the given information and the equation. Distractor B indicates that division is the appropriate computation; however, it provides 2.3 as an incorrect value for 2 hours and 30 minutes. Distractor C presents an incorrect operation, multiplication, although the expression for the number of hours is correct. Distractor D presents multiplication as the operation and also presents an incorrect value for 2 hours and 30 minutes.

Strand	<b>Mathematical Reasoning (MR)</b>	<p><b>If <math>n</math> is any odd number, which of the following is true about <math>n + 1</math>?</b></p> <p><b>A</b> It is an odd number.</p> <p><b>B</b> It is an even number.</p> <p><b>C</b> It is a prime number.</p> <p><b>D</b> It is the same number as <math>n - 1</math>.</p>
Standard	<b>MR1.2</b>	
	<b>Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.</b>	
Constructs	<b>Conceptual Understanding, Problem Solving</b>	

The challenge for many students in formulating mathematical conjectures is precision of language. CAHSEE test questions for this standard focus on both of its components: formulation of a conjecture and justification of a conjecture. Students may be asked to make conjectures based on indirect or incomplete evidence. Test questions may state a conjecture and ask students to choose among reasons that the conjecture is reasonable.

### *Sample Test Question*

The sample question presents  $n$  as any odd number and then requires evaluation of four conjectures. The correct answer is choice B. Students should reason that if  $n$  is odd,  $n + 1$  is necessarily even. Students may also attempt to find counterexamples, i.e., examples of odd  $n$  and also odd  $n + 1$ . This question is also classified in the Algebra and Functions strand for purposes of reporting student scores.

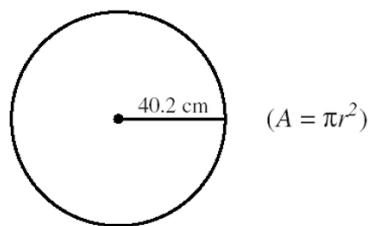
### *Analysis of Distractors*

Students should reason that distractor A is always false because  $n + 1$  must always be even. Distractor C is true only when  $n = 1$ , since 2 is the only even prime. Distractor D is never true, since  $n - 1$  always differs from  $n + 1$  by 2.

Strand **Mathematical Reasoning (MR)**

Standard **MR2.1**  
**Use estimation to verify the reasonableness of calculated results.**

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



Louis calculated the area of the circle above and got an answer of  $50.769 \text{ cm}^2$ . He knew his answer was wrong because the correct answer should be about

- A  $4 \times 4 \times 4 = 64$ .
- B  $3 \times 3 \times 40 = 360$ .
- C  $31 \times 4 \times 4 = 496$ .
- D  $3 \times 40 \times 40 = 4800$ .

Students who have developed the mental habit of estimating and verifying the reasonableness of calculated results will be well prepared to respond to test questions written for standard MR2.1. This standard requires students to use estimating skills in computation and compare estimated results to calculated results in order to judge their reasonableness.

### *Sample Test Question*

The sample test question contains the calculated result  $50.769 \text{ cm}^2$ , and students must choose an estimation strategy that appropriately uses the dimensions of the circle and calculation for the area of a circle. The correct answer is choice D; since  $A = \pi r^2$ ,  $\pi \approx 3$ , and  $40.2 \approx 40$ , the appropriate estimate is  $3 \times 40 \times 40$ . This question is also classified in the Measurement and Geometry strand for purposes of reporting student scores.

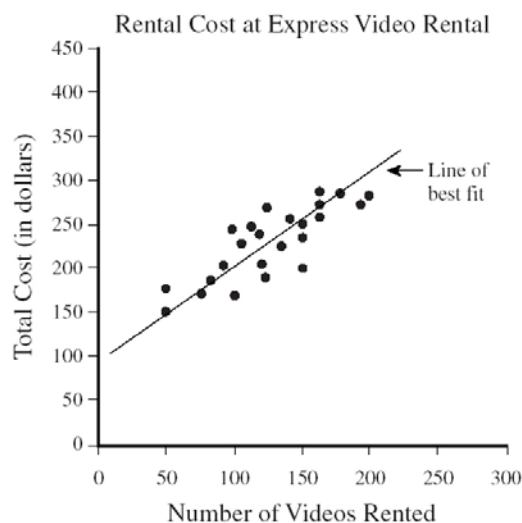
### *Analysis of Distractors*

Distractor A represents an inappropriate selection of estimates for the dimensions of the circle. Distractor B uses the square of  $\pi$ , rather than the radius, as part of the estimate. Distractor C uses 4, rather than 3, as an estimate for  $\pi$ .

Strand **Mathematical Reasoning (MR)**

Standard **MR2.3**  
**Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.**

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



**Using the line of best fit shown on the scatter plot above, which of the following best approximates the rental cost per video to rent 300 videos?**

- A \$3.00
- B \$2.50
- C \$2.00
- D \$1.50

Graphs provide a quick summary of data or functions but may not include the specific information required to answer a particular question. By identifying trends and patterns and using interpolation and extrapolation, students may be able to obtain a reasonable estimate of the needed information. This content standard has two components: estimating graphically and solving for unknown quantities. CAHSEE test questions for the standard may ask students to find or identify the most accurate line of best fit through a scatterplot, to extract information from a graph by interpolation or extrapolation, or to identify an equation that could be used to solve a problem shown in a graph.

### *Sample Test Question*

The sample question includes a scatterplot that relates total cost to number of videos rented. Finding the line of best fit requires an approximation of the correspondence between total cost (y-axis) and videos rented (x-axis). The correct answer is choice D. Students should recognize that extending the line to include an x-value that corresponds to 300 videos allows the interpretation of a corresponding value of the total cost on the y-axis. From the line of best fit, renting 300 videos corresponds to approximately \$450, or \$1.50 per video. This question is also classified in the Probability and Statistics strand for purposes of reporting student scores.

*Analysis of Distractors*

Distractor A requires a total cost of approximately \$900 to obtain a per video cost of \$3.00, which is out of the range of the line of best fit. Distractor B requires a total cost of approximately \$750 to obtain a per video cost of \$2.50, also out of the range of the line of best fit. Similarly, distractor C requires a total cost of approximately \$600 to obtain a per video cost of \$2.00, also out of the range of the line of best fit.

Strand	<b>Mathematical Reasoning (MR)</b>	<p>The winning number in a contest was less than 50. It was a multiple of 3, 5, and 6. What was the number?</p> <p>A 14</p> <p>B 15</p> <p>C 30</p> <p>D It cannot be determined.</p>
Standard	<b>MR2.4</b>	
	<b>Make and test conjectures by using both inductive and deductive reasoning.</b>	
Constructs	<b>Conceptual Understanding, Problem Solving</b>	

Being able to identify patterns (inductive reasoning) and then testing the validity of the patterns (deductive or logical reasoning) are key skills in many fields besides mathematics. This standard requires students to use inductive and deductive reasoning to make and test conjectures. CAHSEE test questions for this standard may require reasoning from general to specific, from specific to general, as well as reasoning by use of counterexample.

#### *Sample Test Question*

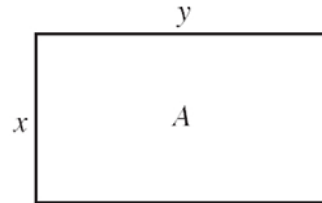
The sample question requires students to use mathematical reasoning to find the winning number in a contest. The correct answer is choice C. Students may begin by finding multiples of 3, 5, and 6 that are less than 50. The multiples of 3 that are less than 50 are 3, 6, 9,...48. The multiples of 5 that are less than 50 are 5, 10, 15,...45. The multiples of 6 that are less than 50 are 6, 12, 18,...48. Thus students should recognize that the only common multiple of 3, 5, and 6 that is less than 50 is 30. This question is also classified in the Number Sense strand for purposes of reporting student scores.

#### *Analysis of Distractors*

Distractor A is the sum of 3, 5, and 6, and distractor B is not a multiple of 6. Students who do not understand the concept of multiples may choose distractor D as the correct answer.

Strand            **Mathematical Reasoning (MR)**  
  
 Standard        **MR3.1**  
**Evaluate the reasonableness of the solution in the context of the original situation.**  
  
 Construct       **Problem Solving**

The rectangle shown below has width  $x$ , length  $y$ , and area  $A$ .



If  $x = 10$  and  $y > 17$ , which of the following cannot be the area of the rectangle?

- A    170
- B    180
- C    190
- D    200

Checking the reasonableness of a solution is an important perspective for students, since calculators and computers are widely available for much routine computation. Content in this standard may include units, signs, or scale in relation to the magnitude described in the problem. Students are asked to determine the reasonableness of answers based on judgments about the characteristics of the quantities within mathematical problems.

### *Sample Test Question*

The sample question presents a rectangle of dimensions 10 and more than 17 units ( $y > 17$ ). The correct answer is choice A. Students should recognize that the area of the rectangle must be more than 170 units, because  $10 \times 17 = 170$  because if the area were equal to 170, the value of  $y$  would be equal to 17, which contradicts  $y > 17$ . This question is also classified in the Measurement and Geometry strand for purposes of reporting student scores.

### *Analysis of Distractors*

Distractors B, C, and D could each be the area of the rectangle, because  $y = 18$ ,  $y = 19$ , and  $y = 20$  do not contradict  $y > 17$ .



Strand            **Mathematical Reasoning (MR)**  
  
 Standard        **MR3.3**  
**Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.**  
  
 Construct        **Problem Solving**

Len runs a mile in 8 minutes. At this rate how long will it take him to run a 26-mile marathon?

Which of the following problems can be solved using the same arithmetic operations that are used to solve the problem above?

- A    Len runs 26 miles in 220 minutes. How long does it take him to run each mile?
- B    A librarian has 356 books to place on 18 shelves. Each shelf will contain the same number of books. How many books can the librarian place on each shelf?
- C    A cracker box weighs 200 grams. What is the weight of 100 boxes?
- D    Each basket of strawberries weighs 60 grams. How many baskets can be filled from 500 grams of strawberries?

One key problem-solving skill is to recognize how a new problem is like a simpler or more familiar problem. This content standard requires students to understand the process by which problems are reasoned, analyzed, and solved. CAHSEE test questions for this standard may include the selection of appropriate analogs to a given problem situation, and the relevance of the analogs as applied to the types of reasoning, patterns of operations, or logical extensions, rather than to context or other more superficial characteristics of the problem. Test questions for this standard may or may not require numerical solutions.

### *Sample Test Question*

The sample test question presents a mathematical problem in context that requires multiplication to find a total. Students must select the most appropriate analog to finding the total time for running 26 miles at the rate of 1 mile per 8 minutes. The correct answer is choice C. Students should recognize the analog in using multiplication to determine the total weight of 100 cracker boxes, with 1 cracker box weighing 200 grams. This question is also classified in the Number Sense strand for purposes of reporting student scores.

### *Analysis of Distractors*

The distractors present division problems. In distractor A, time, 220 minutes, is divided by distance, 26 miles. In distractor B, the number of books, 356, is divided by the number of shelves, 18. In distractor D, the total weight, 500 grams, is divided by the weight of 1 basket, 60 grams.